

## The Event Calculus on High Frequency Finance

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## Ignoring the “Obvious”

- Where can we place a piece without collapsing this pile?
- We should be able to evaluate risks given our physics knowledge
- We ignore the physics in financial markets!



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## The Physics in Markets

- We don't know many things in this pile
  - Pieces could be wet
  - Pieces might stick together
- But clearing rules in a market are designed!
- So we must be able to study its physics!



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## Event Calculus

- Attempt to define market dynamics formally
  - to avoid ambiguity in verbal descriptions
- Markets can be described by states
- Events change the state of the market
- We want to study consequences of events
  - Maintain *consequential closure* if possible
- We want to know exactly what are included in our analysis

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## An Event Calculus

### State + Orders → State

- How should a state be modelled?

State = (Bid\_Q, Offer\_Q)

Bid\_Q = ((P<sub>1</sub>, V<sub>1</sub>), (P<sub>2</sub>, V<sub>2</sub>), ..., (P<sub>bq</sub>, V<sub>bq</sub>))

- Where P<sub>1</sub> > P<sub>2</sub> > ... P<sub>bq</sub>

Offer\_Q = ((P<sub>1</sub>, V<sub>1</sub>), (P<sub>2</sub>, V<sub>2</sub>), ..., (P<sub>oq</sub>, V<sub>oq</sub>))

- Where P<sub>1</sub> < P<sub>2</sub> < ... P<sub>oq</sub>

i.e. Order book + New orders → New State

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## How Orders Are Processed

- We assume that orders are in a queue
- One order is processed at a time
  - Orders = (Order<sub>1</sub>, Order<sub>2</sub>, ..., Order<sub>n</sub>)
  - State + (Order<sub>1</sub>, Order<sub>2</sub>, ..., Order<sub>n</sub>) →  
(State + Order<sub>1</sub>) + (Order<sub>2</sub>, ..., Order<sub>n</sub>)
  - Order = (Order\_Type, Price, Volume)
  - Order\_Type = bid | offer

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### Clearing of a sell order

- Let
  - Bid\_Q1 = ((P1, V1), (P2, V2), ...)
  - Offer\_Q1 = ((P3, V3), (P4, V4), ...)
  - Order1 = (sell, P, V)
- If  $P1 < P$  then (Bid\_Q1, Offer\_Q1) + (sell, P, V) → (Bid\_Q1, Offer\_Q1 ⊕ (sell, P, V))
- If  $P1 ≥ P$  then (Bid\_Q1, Offer\_Q1) + (sell, P, V) → (((P1, V1 - min(V1, V)), (P2, V2), ...), Offer\_Q1) + (sell, P, V - min(V1, V))
- Here ⊕ is the queue joining operator which simply puts the orders in ascending order

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### Example snapshot of the market

	Price	Volume
offer 5	1.65	1,000
offer 4	1.64	2,000
offer 3	1.63	1,500
offer 2	1.62	2,000
offer 1	1.61	3,000
bid 1	1.60	2,500
bid 2	1.59	2,000
bid 3	1.58	2,500
bid 4	1.57	1,500
bid 5	1.56	4,000

- What is the consequence of a market order to sell 5,000 units?
  - 2,500 sold at 1.60
  - 2,000 sold at 1.59
  - 500 sold at 1.58
  - Price dropped by 1.25%\*

\* Assume that last transaction price was 1.60

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### Trader positions matter

	Position	Price	Volume	Margin	Triggered below
Trader 1	long	1.65	4,000	4.00%	1.584
Trader 2	long	1.64	2,000	4.00%	1.574
Trader 3	long	1.64	2,000	5.00%	1.558

- Snapshot of trader positions
- Trader 1 will have to sell should price drop below 1.584, for example
- Different traders have different margins

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### Effect of Margin Call

Market after clearing sell order of 5,000

	Price	Volume
offer 5	1.65	1,000
offer 4	1.64	2,000
offer 3	1.63	1,500
offer 2	1.62	2,000
offer 1	1.61	3,000
bid 1	1.58	2,000
bid 2	1.57	1,500
bid 3	1.56	4,000

- Price at 1.58
- Trader 1's margin exceeded
- Trader 1 has to sell its 4,000 units now\*
- This will push the price down to 1.56
- From 1.60, price dropped by 2.5%

\* Assume market mechanism with complete automation

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### Cascaded Effects

	Price	Volume
offer 5	1.65	1,000
offer 4	1.64	2,000
offer 3	1.63	1,500
offer 2	1.62	2,000
offer 1	1.61	3,000
bid 1	1.56	3,500

- Price at 1.56
- Trader 2's margins exceeded now
- Trader 2 has to sell its 2,000 units (at 1.56)
- Lesson:
  - Price drop depends on trader positions
- What happens if there are no buyers left?

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### Revised Model

**State + Orders → State**

- A state is described by the order book plus the trader positions
  - State = (Bid\_Q, Offer\_Q, Trader\_positions)
- If we know the order book plus the trader positions, we can work out the consequential closure
- See Demo

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## Consequences of margin calls

- A trader is in long position because he/she thinks that price will go **up**
- Margin calls cause selling, which help to push the price **down**
  - Exactly the opposite of what he/she expects!
- Cascaded margin calls could lead to substantial falls in price

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## Remarks

- We are not trying to predict
  - We are just studying the consequences of events under our calculus
- Consequences of an order is non-trivial!
- Given a snapshot of the market, including the trader positions, we can ask:
  - ? How big an order would cause the price to go down (up) by, say, 2.5%?
- Useful for assessing Value-at-risk
- Related work: [Liquidity Risk](#)

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## Future Work

- Real markets are far more complex
- What about market making?
  - How should the market maker [review its prices](#) in response to a big purchase?
- We can define new events
  - [Directional changes](#)
- Wiki-style repository of programs
  - To enable [global collaborative research in finance](#)

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## Conclusions

- Top-down classical economics challenged
- Use of physical time questionable
- Intrinsic time is more meaningful
- An **event calculus** helps to identify relevant components and study the physics of markets
  - Cascading effects can be analysed
  - New perspective to Value-at-risk

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