This can be considered as a least squares problem, i.e., to minimise the function

$$E = \sum (a \star x_i + b \star y_i + c \star z_i + d)^2 \quad (5)$$

Straightforward computation yields

$$2 \star \sum (a \star x_i + b \star y_i + c \star z_i + d) = 0 \quad (6)$$

$$d = -(a \star \bar{x} + b \star \bar{y} + c \star \bar{z}) \quad (7)$$

$$E = \sum (a \star \Delta x_i + b \star \Delta y_i + c \star \Delta z_i)^2 \quad (8)$$

$$W = \left( \begin{array}{ccc}
\sum \Delta x_i^2 & \sum \Delta x_i \star \Delta y_i & \sum \Delta x_i \star \Delta z_i \\
\sum \Delta y_i \star \Delta x_i & \sum \Delta y_i^2 & \sum \Delta y_i \star \Delta z_i \\
\sum \Delta z_i \star \Delta x_i & \sum \Delta z_i \star \Delta y_i & \sum \Delta z_i^2
\end{array} \right)$$

It is known that when \((a, b, c)\) is chosen to be the smallest eigenvector of the matrix \(W\), \(E\) takes its minimum value, i.e., the smallest eigenvalue of \(W\).

So by simply doing an eigenvalue decomposition of matrix \(W\), we can get parameters \(a, b, c\). From equation (7), we can also get \(d\).

There may be some observations which are not correct. In order to get the best fit of the predicted ground disparity map, we discard incorrect observations according to its covariance as follows:

After obtaining \(a, b, c\) through eigenvalue decomposition, we compute the covariance for each observation

$$EZ_i = (a \star x_i + b \star y_i + c \star z_i + d)^2 \quad (9)$$

If \(EZ_i > T\), we discard this observation, here \(T\) is the threshold.

A new set of \(a, b, c,\) and \(d\) is computed using the remaining observations and this process is repeated until no observation can be discarded.

3. Ground disparity computation when the head elevates

From the analysis of the ground disparity plane, the coefficients of the plane equation form sine and cosine relation with the head elevation angle. In order to check the feasibility of using this kind of plane, we capture a set of image pairs with different head elevation angles and fit there plane parameters according to their matches. The results shown in Figure 3 show that this is true.

A look-up table is then set for three parameters of the ground disparity plane as functions of the head elevation angle according to these experiment results using spline interpolation.

![Figure 3: Parameters of ground disparity plane d=ax+by+c; (a) parameter b; (b) parameter c.](image)

After obtaining stereo matches, we can compute the average x-coordinate in the cyclopean image and the average disparity for each vertical edge string. From the head state we know the elevation angle, and so the ground disparity plane parameters can be found from the look-up table. Comparing the disparity of the vertical edge and the ground disparity at that position, if the difference is larger than a threshold, then the vertical edge is from an object standing on the ground; it is an obstacle.

2.4 Object tracking

Since we use parallel camera geometry, we will just control head pan and elevation angle to fixate the tracked vertical edge in the middle of the cyclopean image. After we get the correspondence between left and right images, we can compute the disparity and its coordinate in the cyclopean image. Then, from projective geometry, the difference between this coordinate and the centre of the image can be taken as the input signal of the head motion controller. A simple proportional controller is adopted. From Figure 4, the pan angle that the head needs to move through according to the image position of the tracked vertical edge is:

$$\delta \theta = -\tan^{-1}(\frac{x}{f}) \quad (10)$$

$$\theta_d(k) = \theta(k - 1) + \delta \theta \quad (11)$$

where \(x\) is the relative pixel distance of the vertical edge to the middle of the cyclopean image, \(f\) is the focal length in pixel units of the camera. \(\theta(k - 1)\) is the head pan angle feedback at the last sample time. \(\theta_d(k)\) is the demand head pan angle for this sample time.

In order to include the target speed into the controller, we use \(\theta(k) - \theta(k - 1)\) as a prediction of the target speed of the next sample time, so the whole controller becomes:

$$\theta_d(k) = \theta(k) + \delta \theta \quad (12)$$

To keep the head tracking the object standing on the ground, two can be used:

1. move the tracking window down and up as the object becomes close and far away;