Multi-criteria Decision Aid in Financial Decision Making:
Methodologies and Literature Review

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ABSTRACT
Over the past decades the complexity of financial decisions has increased rapidly, thus highlighting the importance of
developing and implementing sophisticated and efficient quantitative analysis techniques for supporting and aiding
financial decision making. Multi-criteria decision aid (MCDA), an advanced field of operations research, provides
financial decision makers (DMs) and analysts a wide range of methodologies, which are well suited to the complexity
of financial decision problems. The aim of this paper is to provide an in-depth presentation of the contributions of
MCDA in the field of finance, focusing on the methods used and their real-world applications. Copyright © 2003
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KEY WORDS: multi-criteria decision aid; finance; multiobjective optimization; outranking relations; preference
disaggregation analysis; applications

1. INTRODUCTION
During the last decades the globalization of financial markets, the intensifying competition
among firms, financial institutions and organizations as well as the rapid economic, social and
technological changes, have led to an increasing uncertainty and instability in the financial and
business environments. Within this new context the importance of making efficient financial
decisions has increased, and the complexity of the financial decision making process has also
increased. This complexity is clearly evident in the existing variety and volume of new financial
products and services.

In this ‘new’ reality, financial researchers and practitioners acknowledge the requirement to
address financial decision problems through integrated and realistic approaches based on sophis-
ticated quantitative analysis techniques. Thus, the connection between the financial theory and the
mathematical modelling become apparent. Techniques from the field of optimization, stochastic

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The multidimensional nature of financial decisions has already been emphasized by researchers who noted the necessity to address financial problems in a broader, more realistic context considering all the pertinent factors involved (Jacquillat, 1972; Zeleny, 1977, 1982; Colson and Zeleny, 1979; Bhaskar and McNamee, 1983; Spronk and Hallerbach, 1997).

Such findings and critical thoughts have already motivated several researchers to explore the potentials of MCDA in addressing financial decision-making problems. The objective of this paper is to provide a state-of-the-art comprehensive review of the research made up to date on the use of MCDA in financial decisions. Initially, Section 2 justifies the multi-criteria nature of financial decisions. Section 3 provides an outline of the MCDA paradigm, whereas Section 4 reviews the applications of MCDA in several fields of finance. Finally, Section 5 concludes the paper and discusses some possible future research directions on the implementation of MCDA in financial institutions and firms.

2. THE MULTI-CRITERIA CHARACTER OF FINANCIAL DECISIONS

Traditional optimization, statistical and econometric analysis approaches used within the financial engineering context are often based on the assumption that the considered problem is well posed, well formulated regarding the reality involved and they usually consider the existence of a single objective, evaluation criterion or point of view that underlies the conducted analysis (i.e. monocriteria paradigm). In such a case the solution of financial problems is easy to obtain. But in reality, the modelling of financial problems is based on a different kind of logic taking into consideration the following elements (i.e. multicriteria paradigm, cf. Roy, 1988):

- The existence of multiple criteria.
- The conflicting situation between the criteria.
- The complex, subjective and ill-structured nature of the evaluation process.
- The introduction of financial decision makers in the evaluation process.

Financial and operation researchers have recently started to adopt this innovative, comprehensive and realistic perspective, which overcomes the restrictive framework of optimization (Zopounidis, 1990, 1995). For example, in capital budgeting decision making, Bhaskar and McNamee (1983) pose the following questions: (a) In assessing investment proposals, do the decision makers have a single objective or multiple objectives? (b) If the decision makers do have multiple objectives, which are those and what is the priority structure of the objectives? In another similar study, Bhaskar (1979) notes that microeconomic theory has largely adopted a single objective function, which is the principle of utility maximization for consumers and profit maximization for firms. The same author presents three categories of criticism regarding the use of this single objective function principle for firms: (a) there exist alternatives to the profit maximization approach which are based on equally simple hypotheses and can better explain reality; (b) the profit maximization or any other equally simple hypothesis is too naive to explain the complex process of decision making; (c) the real-world firms do not have suitable information to enable them to maximize their profits. Furthermore, several other theories of the firm have been postulated and have proposed different objectives than that of the traditional microeconomic theory. One can cite the revenue maximizing model (Baumol, 1959), the manager's utility model (Williamson, 1964), the satisfying model (Simon, 1957) and the behavioural models (Cyert and March, 1963).

On the basis of the above remarks it is possible to distinguish three main reasons which have motivated a change of view in the modelling of the financial problems (Zopounidis, 1999).

- Formulating the problem in terms of seeking the optimum, financial decision makers (i.e. financial analysts, portfolio managers, investors, etc.) get involved in a very narrow problematic, often irrelevant to the real decision problem.
- The different financial decisions are taken by the humans (i.e. financial managers) and not by the models; the decision makers get more and more deeply involved in the decision-making process. In order to solve problems, it becomes necessary to take into consideration their preferences, their experiences and their knowledge.
For financial decision problems such as the choice of investment projects, the portfolio selection, the evaluation of business failure risk, etc., it is an illusion to speak about optimality since multiple criteria must be taken into consideration.

### 3. THE MCDA PARADIGM

Within a multi-criteria context, decision-making problems are realized in the following paradigm: a DM considers a set of alternatives (e.g. firms, investment projects, portfolios, credit applications, etc.) and seeks to take an ‘optimal’ decision considering all the factors that are relevant to the analysis. Since these factors usually lead to conflicting results and conclusions, the ‘optimal' decision is not really optimal in the traditional optimization perspective. Instead, it is a satisfactory non-dominated decision (i.e. a decision that is in accordance with the DM’s system of values and is not dominated by other possible decisions).

Decisions made within this context may be expressed in different forms, which are referred as 'problematics' (Roy, 1996): (1) problematic \( \alpha \): Choosing one alternative, (2) problematic \( \beta \): Sorting the alternatives in homogenous groups defined in a preference order, (3) problematic \( \gamma \): Ranking the alternatives from the best one to the worst one, and (4) problematic \( \delta \): Describing the alternatives in terms of their performance on the criteria. The selection of an investment project is a typical example of a financial decision-making problem where problematic \( \alpha \) (choice) is applicable. The prediction of business failures is an example of problematic \( \beta \) (classification of firms as healthy or failed), the comparative evaluation and ranking of stocks according to their financial and stock market performance is an example of problematic \( \gamma \), whereas the description of the financial characteristics of a set of firms is a good example of problematic \( \delta \). The selection of one of these problematics depends solely on the objective of the analysis and the decision-making context.

The first issue that needs to be addressed within this decision-making context involves the identification of the set of alternatives. This set may have two different forms: continuous or discrete. In the continuous case, \( A \) is assumed to include an infinite number of alternatives (or at least a very large number of alternatives such that their enumeration is very difficult). Resource allocation problems are a typical example of this case. For instance, during the construction of a portfolio of \( n \) securities, it is impossible to identify all the portfolios that can be constructed. However, it is possible to define the efficient set consisting of portfolios that meet the investment constraints imposed by the DM. In the discrete case, \( A \) is assumed to include a finite number of clearly identifiable alternatives \( a_1, a_2, \ldots, a_m \). This situation is often met in several financial decisions, such as bankruptcy prediction and credit risk assessment (\( A \): a set of firms), portfolio selection (\( A \): a set of stocks/portfolios), venture capital investments (\( A \): a set of venture capital financing proposals), country risk evaluation (\( A \): a set of countries), etc.

Irrespective of whether the set of alternatives \( A \) is discrete or continuous, making a decision in a multi-criteria context requires the appropriate aggregation of all the pertinent decision factors \( g = (g_1, g_2, \ldots, g_n) \), which are referred to as “evaluation criteria” or simply “criteria”. Formally, a criterion \( g_i \) is a non-decreasing real-valued function that describes an aspect of the global performance of the alternatives and defines how the alternatives are compared to each other, as follows:

\[
g_{ji} > g_{ki} \iff a_j > a_k \quad (a_j \text{ is preferred to } a_k)
g_{ji} = g_{ki} \iff a_j \sim a_k \quad (a_j \text{ is indifferent to } a_k)
\]

where \( g_{ji} \) denotes the performance of alternative \( a_j \) on criterion \( g_i \).

In making a decision within this multi-criteria context the aggregation of the criteria is a crucial process. This aggregation can be performed in many different ways depending on the form of the criteria aggregation model. Within the MCDA field one can distinguish three main forms of aggregation models: (1) outranking relations (relational form), (2) utility functions (functional form), (3) decision rules (symbolic form). The construction of an aggregation model is mainly of interest in the case where \( A \) is discrete. In such a case the alternatives are clearly identifiable and consequently their performance on each criterion can be specified, rather easily. In the case where \( A \) is continuous, however, this is not a straightforward process, simply because it is impossible to identify all the alternatives that are relevant to the analysis. In this case special interactive aggregation techniques have been developed in MCDA to allow the efficient search of the solution space.
These techniques fall within the multiobjective mathematical programming framework. In all cases, the aggregation of the criteria is performed so as to respect the DM’s judgement policy. To ensure that this objective is achieved some information on the preferential system of the DM must be specified, such as the criteria weights, trade-offs, etc. The required preferential information can be specified either through direct procedures in which a decision analyst elicits it directly from the DM, or through indirect procedures in which the DM provides examples of the decisions situations that he faces and the decision analyst analyses them to determine the required preferential parameters which are most consistent with the DM’s global evaluations. The latter approach is known in the MCDA field as ‘preference disaggregation analysis’ (Jacquet-Lagrèze and Siskos, 1982, 1983, 2001).

Appendix A provides a discussion of the main MCDA methodological approaches that have been used in addressing financial decision-making problems.

4. APPLICATIONS IN FINANCIAL DECISIONS

MCDA approaches are well suited for the study of several financial decision-making problems. The diversified nature of the factors (evaluation criteria, objectives and goals) that affect financial decisions, the complexity of the financial, business and economic environments, the subjective nature of many financial decisions, are only some of the features of financial decisions which are in accordance with the MCDA modelling framework. On the basis of these remarks this section reviews the up-to-date applications of MCDA methodologies in several major financial decisions.

4.1. Bankruptcy and credit risk assessment

The assessment of bankruptcy and credit risk have been major research fields in finance for the last three decades. Bankruptcy risk is derived by the failure of a firm to meet its debt obligations to its creditors, thus leading the firm either to liquidation (discontinuity of the firm’s operations) or to a reorganization programme (Zopounidis and Dimitras, 1998). The concept of credit risk is similar to that of bankruptcy risk, in the sense that in both cases the likelihood that a debtor (firm, organization or individual) will not be able to meet its debt obligations to its creditors, is a key issue in the analysis. Credit risk assessment decisions, however, are not simply based on the estimation of this likelihood; furthermore, they take into account the opportunity cost that may occur when a good client (firm or individual) is denied credit. In both cases, the most common approach used to address bankruptcy and credit risk assessment problems is to develop appropriate models that sort/classify the firms or the individuals into predefined groups (problematic β), e.g. classification of firms as bankrupt/non-bankrupt, or as high credit risk firms/low credit risk firms. Statistical and econometric techniques (discriminant analysis, logit and probit analysis, etc.) have dominated this field for several decades, but recently new methodologies have attracted the interest of researchers and practitioners (for a comprehensive review see Dimitras et al., 1996), including several MCDA methods. A representative list of the MCDA evaluation approaches applied in bankruptcy and credit risk assessment is presented in Table I.

4.2. Portfolio selection and management

Portfolio selection and management involves the construction of a portfolio of securities (stocks, bonds, treasury bills, mutual funds, etc.) that maximizes the investor’s utility. This problem can be realized as a two stage process (Hurson and Zopounidis, 1995, 1997): (1) evaluation of the available securities to select the ones that best meet the investor’s preferences, (2) specification of the amount of capital to be invested in each of the securities selected in the first stage. The implementation of these two stages in the traditional portfolio theory is based on the mean-variance approach developed by Markowitz (1952, 1959). Recently, however, the multidimensional nature of the problem has been emphasized by researchers in finance (Jacquillat, 1972), as well as by MCDA researchers (Zeleny, 1977, 1982; Colson and Zeleny, 1979; Spronk and Hallerbach, 1997). Within this multidimensional context, the MCDA paradigm provides a plethora of appropriate methodologies to support the evaluation of the available securities as well as portfolio construction. The former (securities’ evaluation) has been studied by MCDA researchers using discrete evaluation methods (outranking relations, MAUT, PDA, rough sets). Studies conducted on this topic have focused on the modelling and representation of the investor’s policy, goals and objectives in a mathematical model. The model
aggregates all the pertinent factors describing the performance of the securities and provides their overall evaluation. The securities with the higher overall evaluation are selected for portfolio construction purposes in a latter stage of the analysis. The portfolio construction process is realized within the MCDA framework as an MMP/GP problem. The DM specifies the portfolio construction criteria, his objectives/goals and an iterative and interactive process is invoked to identify a portfolio that best meets his investment policy. Table II summarizes several studies involving the application of MCDA approaches in portfolio selection and management.

4.3. Corporate performance evaluation
The evaluation of the performance of corporate entities and organizations is an important activity for their management and shareholders as well as for investors and policy makers. Such an evaluation provides the management and the shareholders with a tool to assess the strength and weakness of the firm as well as its competitive advantages over its competitors, thus providing guidance on the choice of the measures that need to be taken to overcome the existing problems. Investors (institutional and individual) are interested in the assessment of corporate performance for guidance to their investment decisions, while policy makers may use such an assessment to identify the existing problems in the business environment and take measures that will ensure a sustainable economic growth and social stability. The performance of a firm or an organization is clearly multidimensional, since it is affected by a variety of factors of different nature, such as: (1) financial factors indicating the financial position of the firm/organization, (2) strategic factors of qualitative nature that define the internal operation of the firm and its relation to the market (organization, management, market trend, etc.; Zopounidis, 1987), and (3) economic factors that define the economic and business environment. The aggregation of all these factors into a global evaluation index is a subjective process that depends on the DM’s values and judgement policy. These findings are in accordance with the MCDA paradigm, thus leading several operational researchers to the investigation of the capabilities that MCDA methods provide in supporting DMs in making decisions regarding the evaluation of corporate performance. An indicative list of studies on this topic is given in Table III.

Table I. Applications of MCDA approaches in bankruptcy and credit risk assessment

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<th>Approaches</th>
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<td>Multiattribute utility theory</td>
<td>AHP</td>
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<td>Outranking relations</td>
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<td>UTADIS</td>
<td>Zopounidis and Doumpos (1998, 1999a,b)</td>
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<td>MHDIS</td>
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<td>Rough set theory</td>
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4.4. Investment appraisal

In most cases the choice of investment projects is an important strategic decision for every firm, public or private, large or small. Therefore, the process of an investment decision should be conveniently modelled. In general, the investment decision process consists of four main stages: perception, formulation, evaluation and choice. The financial theory intervenes only in the stages of evaluation and choice (Colasse, 1993) based on traditional financial criteria such as the payback period, the accounting rate of return, the net present value, the internal rate of return, the index of profitability, the discounted payback method, etc. This approach, however, entails some shortcomings such as the difficulty in aggregating the conflicting results of each criterion and the elimination of important qualitative variables from the analysis (Zopounidis, 1999). MCDA, on the other hand, contributes in a very original way to the investment decision process, supporting all stages of the investment process. Concerning

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<th>Approaches</th>
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<td>Multiobjective/goal programming</td>
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<td>Leung et al. (2001)</td>
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<td>Multiattribute utility theory</td>
<td>AHP</td>
<td>Saaty et al. (1980)</td>
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<td>MACBETH</td>
<td>Bana e Costa and Scares (2001a,b)</td>
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<td>Other methods</td>
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<td>Outranking relations</td>
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<td>Preference disaggregation</td>
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<td>Rough set theory</td>
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the stages of perception and formulation, MCDA contributes to the identification of possible actions (investment opportunities) and to the definition of a set of potential actions (possible variants, each variant constituting an investment project in competition with others). Concerning the stages of evaluation and choice, MCDA supports the introduction in the analysis of both quantitative and qualitative criteria. Criteria such as the urgency of the project, the coherence of the objectives of the projects with those of the general policy of the firm as well as the social and environmental aspects should be taken into careful consideration. Therefore, MCDA contributes in investment appraisal through the identification of the best investment projects according to the problematic chosen, the satisfactory resolution of the conflicts between the criteria, the determination of the relative importance of the criteria in the decision-making process, and the revealing of the investors’ preferences and system of values. These attractive features have been the main motivation for the use of MCDA methods in investment appraisal in several real-world cases. A representative list of studies is presented in Table IV.

4.5. Other financial decision problems
Except for the above financial decision-making problems, MCDA approaches are applicable in several other sub-fields of finance. Table V lists some additional applications of MCDA methods in other financial problems, including venture capital, country risk assessment, financial planning and the prediction of corporate mergers and acquisitions. In venture capital investment decisions, MCDA methods are used both as tools to evaluate the firms that seek venture capital financing. In country risk assessment, MCDA methods are used to develop models that aggregate the appropriate economic, financial and socio-political factors, to support the evaluation of the creditworthiness and the future prospects of the countries. In financial planning, MCDA methods that employ the MMP/GP framework are employed to construct single and multi-period financial planning strategies for firms and organizations that meet the objectives of the top management of the firms. Finally, in corporate mergers and acquisitions MCDA methods are used to assess the likelihood that a firm will be merged or acquired on the basis of financial information (financial ratios) and strategic factors.

5. CONCLUSIONS AND FUTURE PERSPECTIVES
This paper discussed the contribution of the MCDA in financial decision-making problems, focusing on the justification of the multidimen-
sional character of financial decisions and the use of different MCDA methodologies to support them.

Overall, the main advantages that the MCDA paradigm provides in financial decision making, could be summarized in the following aspects (Zopounidis, 1999): (1) the possibility of structuring complex evaluation problems, (2) the introduction of both quantitative (i.e. financial ratios) and qualitative criteria in the evaluation process, (3) the transparency in the evaluation, allowing good argumentation in financial decisions, and (4) the introduction of sophisticated, flexible and realistic scientific methods in the financial decision making process.
decision-making process. But most important of all, MCDA enables the DM to participate actively in the financial decision-making process and supports him in understanding the peculiarities and the special features of the real-world problem that he faces. Thus, the DM is not restricted to a passive role implementing automatically the “optimal” solutions obtained from a mathematical model. Instead, he participates in the model formulation process as well as in the analysis and implementation of the results, according to his judgement policy.

In conclusion, MCDA methods seem to have a promising future in the field of financial management, because they offer a highly methodological and realistic framework to decision problems. Nevertheless, their success in practice depends heavily on the development of computerized multi-criteria decision support systems. Financial institutions as well as firms acknowledge the multi-dimensional nature of financial decision problems (Bhaskar and McNamee, 1983). Nevertheless, they often use optimization or statistical approaches to address their financial problems, since optimization and statistical software packages are easily available in relatively low cost, even though many of these software packages are not specifically designed for financial decision-making problems. Consequently, the use of MCDA methods to support real-time financial decision-making, calls upon the development of integrated user-friendly multi-criteria decision support systems that will be specifically designed to address financial problems. Examples of such systems are the CGX system (Srinivasan and Ruparel, 1990), the BANKS system (Mareschal and Mertens, 1992), the BANKADVISER system (Mareschal and Brans, 1991), the INVEX system (Vranes et al., 1996), the FINEVA system (Zopounidis et al., 1996), the FINCLUS system (Zopounidis and Doumpos, 1998), the INVESTOR system (Zopounidis and Doumpos, 2000b), etc. The development and promotion of such systems is a key issue in the successful application of MCDA methods in finance.

APPENDIX A: OUTLINE OF MCDA METHODOLOGIES

A.1. Multiobjective mathematical programming
Multiobjective mathematical programming (MMP) is an extension of the traditional mathematical programming theory in the case where multiple objective functions need to be optimized. The general formulation of an MMP problem is as follows:

Max/Min\{g_1(x), g_2(x), \ldots, g_n(x)\}

Subject to: \( x \in B \)

where \( x \) is the vector of the decision variables. \( g_1, g_2, \ldots, g_n \) are the objective functions (linear or non-linear) to be optimized and \( B \) is the set of feasible solutions.

In contrast to the traditional mathematical programming theory, within the MMP framework the concept of optimal solution is no longer applicable. This is because the objective functions are of conflicting nature (the opposite is rarely the case). Therefore, it is not possible to find a solution that optimizes simultaneously all the objective functions. In this regard, within the MMP framework the major point of interest is to search for an appropriate ‘compromise’ solution. In searching for such a solution only the efficient set needs to be considered. The efficient set consists of solutions which are not dominated by any other solution on the pre-specified objectives. Such solutions are referred to as efficient solutions, non-dominated solutions or Pareto optimal solutions.

Several appropriate solution procedures have been developed to solve MMP problems. These procedures are interactive and iterative. The general framework within which these procedures operate can be considered as a two stage process. In the first stage an initial efficient solution is obtained and it is presented to the DM. If this solution is considered acceptable by the DM (i.e. if it satisfies his expectations on the given objectives), then the solution procedure stops. If this is not the case, then the DM is asked to provide information regarding his preferences on the pre-specified objectives. This information involves the objectives that need to be improved as well as the trade-offs that he is willing to undertake to achieve these improvements. The objective of defining such information is to specify a new search direction for the development of new improved solutions. This process is repeated until a solution is obtained that is in accordance with the DM’s preferences, or until no further improvement of the current, solution is possible. Some well-known methodologies that operate within the above general framework for addressing MMP problems include the methods developed by Benayoun et al. (1971),

An alternative approach to address constrained optimization problems in the presence of multiple objectives, is the goal programming (GP) approach, founded by Charnes and Cooper (1961). The concept of goal is different from that of objective. An objective simply defines a search direction (e.g. profit maximization). On the other hand, a goal defines a target against which the attained solutions are compared (Keeney and Raiffa, 1993). In this regard, GP optimizes the deviations from the pre-specified targets, rather than the performance of the solutions. The general form of a GP model is the following:

\[
\begin{align*}
\text{Max/Min } & h(d_i^+, d_i^-) \\
\text{Subject to: } & \sum_{i=1}^{b} f_i(x) + d_i^+ - d_i^- = c_i, \\
& x \in B, \\
& d_i^+, d_i^- \geq 0
\end{align*}
\]

where \( f_i \) is goal \( i \) defined as a function (linear or non-linear) of the decision variables \( x \), \( c_i \) is the target value for goal \( f_i \), \( d_i^+ \) and \( d_i^- \) are the deviations from the target value \( c_i \) (\( d_i^+ \cdot d_i^- = 0 \)), representing the under-achievement and over-achievement of the goal, respectively, \( h \) is a function (usually linear) of the deviational variables.

The above general formulation shows that actually an objective function of an MMP formulation is transformed into a constraint within the context of a GP formulation. The right-hand size of these constraints includes the target values of the goals, which can be defined either as some satisfactory values of the goals or as their optimal values. The simplicity of GP formulations has been the main reason for their wide popularity among researchers and practitioners. Spronk (1981) provides an extensive discussion of GP as well as its applications in the field of financial planning.

A.2. Outranking relations

The foundations of the outranking relation theory have been set by Bernard Roy (Roy, 1968) during the late 1960s through the development of the ELECTRE family of methods (ELimination Et Choix Traduisant la REalité). Since then, it has been widely used by MCDA researchers, mainly in Europe.

The outranking relation is a binary relation that enables the DM to assess the strength of the outranking character of an alternative \( a_j \) over an alternative \( a_k \). This strength increases if there are enough arguments (coalition of the criteria) to confirm that \( a_j \) is at least as good as \( a_k \), while there is no strong evidence to refuse this statement.

Outranking relations techniques operate into two stages. The first stage involves the development of an outranking relation among the considered alternatives, while the second stage involves the exploitation of the developed outranking relation to choose the best alternatives (problematic \( \pi \)), sort them into homogenous groups (problematic \( \beta \)) or rank them from the most to the least preferred ones (problematic \( \gamma \)).

Some of the most widely known outranking relations methods include the family of the ELECTRE methods (Roy, 1991) and the family of the PROMETHEE methods (Brans and Vincke, 1985; Brans et al., 1986). These methods are briefly discussed below. A detailed presentation of all outranking methods can be found in the books of Vincke (1992) and Roy and Bouyssou (1993).

A.3. The ELECTRE methods

The family of ELECTRE methods has been initially introduced by Roy (1968), through the development of the ELECTRE I method, the first method to employ the outranking relation concept. Since then several extensions have been proposed, including ELECTRE II, III, IV, IS and TRI (Roy, 1991). These methods address different types of problems, including choice (ELECTRE I, IS), ranking (ELECTRE II, III, IV) and sorting/classification (ELECTRE TRI).

Given a set of alternatives \( A = \{a_1, a_2, \ldots, a_m\} \) any of the above ELECTRE methods can be employed depending on the objective of the analysis (choice, ranking, sorting/classification). Despite their differences, all the ELECTRE methods are based on the identification of the strength of affirmations of the form \( F = \text{‘alternative } a_j \text{ is at least as good as alternative } a_k \text{’} \). The specification of this strength requires the consideration of the arguments that support the affirmation \( F \) as well as the consideration of the arguments that are against it. The strength of the arguments that support \( F \) is analysed through the ‘concordance test’. The measure used to assess this strength is the global concordance index \( C(a_j, a_k) \in [0, 1] \). The closer is \( C \)
to unity, the higher the strength of the arguments that support the affirmation $F$. The concordance index is estimated as the weighted average of partial concordance indices defined for each criterion:

$$C(a_j, a_k) = \sum_{i=1}^{n} w_i c_i(g_{ji} - g_{ki})$$

where $W_i$ is the weight of criterion $g_i$ ($\Sigma w_i = 1$, $w_i \geq 0$) and $c_i(g_{ji} - g_{ki})$ is the partial concordance index defined as function of the difference $g_{ji} - g_{ki}$ between the performances of $a_j$ and $a_k$ on criterion $g_i$. The partial concordance index measures the strength of the affirmation $F' = a_j$ is at least as good as $a_k$ on the basis of criterion $g_i$. The partial index is normalized in the interval [0, 1], with values close to 1 indicating that $F'$ is true and values close to 0 indicating that $F'$ is false.

Except for assessing the strength of the arguments that support the affirmation $F$, the strength of the arguments against $F$ is also assessed. This is performed through the “discordance test”, which leads to the calculation of the discordance index $D_i(g_{ji} - g_{ki})$ for each criterion $g_i$. Conceptually, the discordance index $D_i(g_{ji} - g_{ki})$ measures the strength of the indications against the affirmation $F'$. The higher the discordance index the more significant is the opposition of the criterion on the validity of the affirmation $F$. If the strength of this opposition for criterion $g_i$ is above a critical level (veto threshold), then the criterion vetoes the validity of the affirmation $F$ irrespective of the performance of the considered pair of alternatives $(a_j, a_k)$ on the other criteria.

Once the concordance and discordance tests are performed, their results (concordance index $C$, discordance indices $D_i$) are combined to construct the final outranking relation. The way that this combination is performed, as well as the way that its results are employed to choose, rank or sort the alternatives depends on the specific ELECTRE method that is used. Details on these issues can be found in the works of Roy (1991, 1996) as well as in the book of Roy and Bouyssou (1993).

A.4. The PROMETHEE methods

The development of the PROMETHEE (Preference Ranking Organization Method of Enrichment Evaluations) family of methods began in the mid-1980s with the work of Brans and Vincke (1985) on the PROMETHEE I and II methods.

The PROMETHEE method leads to the development of an outranking relation that can be used either to choose the best alternatives (PROMETHEE I) or to rank the alternatives from the most preferred to the least preferred ones (PROMETHEE II). For a given set of alternatives $A$, the evaluation process in PROMETHEE involves performing all pairwise comparisons $(a_j, a_k)$ to determine the preference index $\pi(a_j, a_k)$ measuring the degree of preference for $a_j$ over $a_k$, as follows:

$$\pi(a_j, a_k) = \sum_{i=1}^{n} w_i P_i(g_{ji} - g_{ki}) \in [0, 1]$$

The preference index is similar to the global concordance index of the ELECTRE methods. The higher the preference index (closer to unity), the higher the strength of the preference for $a_j$ over $a_k$. The calculation of the preference index depends on the specification of the criteria weights $w_i(\Sigma w_i = 1, w_i \geq 0)$ and the preference functions $P_i$ for each criterion $g_i$. The preference functions are increasing functions of the difference $g_{ji} - g_{ki}$ between the performances of $a_j$ and $a_k$ on criterion $g_i$. The preference functions are normalized between 0 and 1. The case $P_i(a_j, a_k) \approx 1$ indicates a strong preference for $a_j$ over $a_k$ in terms of the criterion $g_i$, whereas the case $P_i(a_j, a_k) \approx 0$ indicates weak preference. Generally, the preference functions may have different forms, depending on the judgement policy of the DM. Brans and Vincke (1985) proposed six specific forms (generalized criteria) which seem sufficient in practice.

The results of the comparisons made for all pairs of alternatives $(a_j, a_k)$ are organized in a graph (value outranking graph). The nodes of the graph represent the alternatives under consideration, whereas the arcs between nodes $a_j$ and $a_k$ represent the preference of alternative $a_j$ over $a_k$ (if the direction of the arc is $a_j \rightarrow a_k$) or the opposite (if the direction of the arc is $a_k \rightarrow a_j$). Each arc is associated with a flow representing the preference index $\pi(a_j, a_k)$. The sum of all flows leaving a node $a_j$ is called the leaving flow $\phi^-(a_j)$. The leaving flow provides a measure of the outranking character of alternative $a_j$ over all the other alternatives in $A$. In a similar way, the sum of all flows entering a node $a_j$ is called the entering flow $\phi^+(a_j)$. The entering flow measures the outranked character of alternative $a_j$ compared to all the other alternatives in $A$. 

On the basis of these flows the heuristic procedures of PROMETHEE I and II are employed to choose the best alternatives (PROMETHEE I) or to rank the alternatives from the most preferred to the least preferred ones (PROMETHEE II). The choice of the best alternatives in the PROMETHEE I method involves the definition of the preference \( P \) and indifference \( I \) relations of the basis of the leaving and entering flows of the outranking graph (for details see Brans and Vincke, 1985). The ranking of the alternatives in the PROMETHEE II method is based on the difference between the leaving and the entering flow \( \phi(a_j) = \phi^+(a_j) - \phi^-(a_j) \), which provides the net flow for the node (alternative) \( a_j \). The net flow constitutes the overall evaluation index of the performance and ranking of the alternatives. The most preferred alternatives are the ones with the higher net flows, whereas the alternatives with the lower net flows are considered as the least preferred ones.

### A.5. Utility functions-based approaches

The multiattribute utility theory (MAUT; Keeney and Raiffa, 1993) extends the traditional utility theory to the multidimensional case. Even from the early stages of the MCDA field, the strong theoretical foundations of the MAUT framework have been among the cornerstones of the development of MCDA and its practical implementation. The objective of MAUT is to model and represent the DM’s preferential system into a utility/value function \( U \). The utility function is a non-linear function defined on the criteria space, such that

\[
U(a_j) > U(a_k) \iff a_j \succ a_k \quad (a_j \text{ is preferred to } a_k)
\]

\[
U(a_j) = U(a_k) \iff a_j \sim a_k \quad (a_j \text{ is preferred to } a_k)
\]

The most commonly used form of utility function is the additive one:

\[
U(a_j) = p_1u_1(g_{j1}) + \cdots + p_nu_n(g_{jn})
\]

where, \( u_1, u_2, \ldots, u_n \) are the marginal utility functions corresponding to the evaluation criteria. Each marginal utility function \( u_i(g_i) \) defines the utility/value of the alternatives for each individual criterion \( g_i \). The constants \( p_1, p_2, \ldots, p_n \) are weights of the criteria and they are defined such that they sum-up to one.


Generally, the process for developing an additive utility function is based on the cooperation between the decision analyst and the DM. This process involves the specification of the criteria trade-offs and the form of the marginal utility functions. The specification of these parameters is performed through interactive procedures, such as the mid-point value technique proposed by Keeney and Raiffa (1993). The realization of such interactive procedures is often facilitated by the use of multi-criteria decision support systems, such as the MACBETH system developed by Bana e Costa and Vansnick (1994).

However, the implementation of such interactive procedures in practice can be cumbersome, mainly because it is rather time consuming and depends on the willingness of the DM to provide the required information and the ability of the decision analyst to elicit it efficiently. The preference disaggregation approach of MCDA (PDA; Jacquet-Lagrèze and Siskos, 1982, 1983) provides the methodological framework to cope with this problem. PDA refers to the analysis (disaggregation) of the global preferences (judgement policy) of the DM in order to identify the criteria aggregation model that underlies the preference result (ranking or classification/sorting). Similarly to MAUT, PDA uses common utility decomposition forms to model the DM’s preferences. Nevertheless, instead of employing a direct procedure for estimating the global utility model (MAUT), PDA uses regression-based techniques (indirect estimation procedure). More specifically, in PDA the parameters of the utility decomposition model are estimated through the analysis of the DM’s overall preference on some reference alternatives \( A' \), which may include either examples of past decisions or a small subset of the alternatives under consideration. The DM is asked to provide a ranking or a classification of the reference alternatives according to his decision policy (global preferences). Then, using regression-based techniques the global preference model is estimated so that the DM’s global evaluation is reproduced as consistently as possible by the model. A comprehensive bibliography on prefer-
ence disaggregation methods can be found in the works of Jacquet-Lagrèze and Siskos (1982, 1983, 2001) and Pardalos et al. (1995).

PDA methods are particularly useful in addressing financial decision-making problems (Zopounidis, 2001). The repetitive character of financial decisions and the requirement for real-time decision support are two features of financial decisions which are consistent with the PDA framework. Thus, several PDA methods have been extensively used in addressing financial decision problems, mainly in cases where a ranking or sorting/classification of the alternatives is required. The following subsections provide a brief description of some representative PDA methods which have been used in financial problems.

A.6. The UTA method
The UTA method (UTilités Additives; Jacquet-Lagrèze and Siskos, 1982) is an ordinal regression method developed to address ranking problems. The objective of the method is to develop an additive utility function which is as consistent as possible with the DM’s judgement policy. The input to the method involves a set of reference alternatives \( A' = \{a_1, a_2, \ldots, a_m\} \). For each reference alternative the DM is asked to provide his global evaluation so as to form a total pre-order of the alternatives in \( A' \). The developed utility model is assumed to be consistent with the DM’s judgement policy if it is able to reproduce the given pre-order of the alternatives in \( A' \). The objective of the method is to develop an additive utility model that is consistent with the DM’s judgement policy if and only if the following condition is satisfied:

\[
U(a_j) \geq U(a_i) \quad \forall a_j \in A_i
\]

In developing the utility model to meet this requirement, there are two types of possible errors which may occur (Siskos and Yannacopoulos, 1985): (1) the under-estimation error \( \sigma_j^- \) when the developed model assigns an alternative \( a_j \in A' \) to a lower rank than the one specified in the given pre-order (the alternative is under-estimated by the DM), and (2) the over-estimation error \( \sigma_j^+ \) when the developed model assigns an alternative \( a_j \in A' \) to a higher rank than the one specified in the given pre-order (the alternative is over-estimated by the DM). The objective of the model development process is to minimize the sum of these errors. This is performed through linear programming techniques (Jacquet-Lagrèze and Siskos, 1982).

A.7. The UTADIS method
The UTADIS method (UTilités Additives DIScriminantes; Jacquet-Lagrèze, 1995; Dourmpou and Zopounidis, 2002) is a variant of the UTA method, developed for sorting/classification problems. Similarly to the UTA method, the DM is asked to provide his global evaluation on a set of reference alternatives \( A' = \{a_1, a_2, \ldots, a_m\} \). In this case, however, the DM is not asked to rank the alternatives in \( A' \). Instead, he classifies the reference alternatives into homogenous groups \( C_1, C_2, \ldots, C_q \) defined in an ordinal way, such that \( C_1 > C_2 > \cdots C_q \) (i.e. group \( C_1 \) includes the most preferred alternatives, whereas group \( C_q \) includes the least preferred ones). Within this context, the developed additive utility model will be consistent with the DM’s global judgement if the following conditions are satisfied:

\[
\begin{align*}
U(a_j) &\geq t_1, & \forall a_j &\in C_1 \\
t_1 &> U(a_j) \geq t_2, & \forall a_j &\in C_2 \\
\vdots & & \vdots & \\
U(a_j) &< t_{q-1}, & \forall a_j &\in C_q
\end{align*}
\]

where \( t_1 > t_2 > \cdots > t_{q-1} \) are thresholds defined in the global utility scale \([0,1]\) to discriminate the groups (each \( t_k \) is the lower bound of group \( C_k \)). Similarly, to the UTA method, the under-estimation and over-estimation errors are also used in the UTADIS method to measure the differences between the model’s results and the predefined classification of the reference alternatives. In this case, the two types of errors are defined as follows: (1) the over-estimation error \( \sigma_j^- = \max\{0, t_k - U(a_j)\}, \forall a_j \in C_k, k = 1, 2, \ldots, q - 1 \), (2) the under-estimation error \( \sigma_j^+ = \max\{0, U(a_j) - t_k\}, \forall a_j \in C_k, k = 2, 3, \ldots, q \). The additive utility model is developed to minimize these errors using a linear programming formulation (for details see Dourmpou and Zopounidis, 2002).

Recently, several new variants of the original UTADIS method have been proposed (UTADIS I, II, III) to consider different optimality criteria during the development of the additive utility classification model (Zopounidis and Dourmpou, 1997; Dourmpou and Zopounidis, 2002).

A.8. The MHDIS method
The MHDIS method (Multi-group Hierarchical Discrimination: Zopounidis and Dourmpou, 2000a) extends the PDA framework of the UTADIS method in complex sorting/classification problems.
involving multiple-groups (of course the method is also applicable in the simple two-group case). As the name of the method implies, MHDIS addresses sorting problems through a hierarchical procedure, during which the groups are distinguished progressively, starting by discriminating group $C_1$ (most preferred alternatives) from all the other groups \{$C_2,C_3,\ldots,C_q$\}, and then proceeding to the discrimination between the alternatives belonging to the other groups. At each stage of this sequential/hierarchical process two additive utility functions are developed for the classification of the alternatives. Assuming that the classification of the alternatives should be made into $q$ ordered classes $C_1>C_2>\cdots>C_q$, $2(q-1)$ additive utility functions are developed. These utility functions have the following additive form:

$$U_k(g) = \sum_{i=1}^{n} u_{ik}(g_i), \quad U_{-k}(g) = \sum_{i=1}^{n} u_{-ki}(g_i)$$

Both functions are defined between 0 and 1. The function $U_k$ measures the utility for the DM of a decision to assign an alternative into group $C_k$, whereas the second function $U_{-k}$ corresponds to the classification into the set of groups $C_{-k} = \{C_{k+1},C_{k+2},\ldots,C_q\}$. The rules used to perform the classification of the alternatives are the following:

If $U_1(a_j) > U_{-1}(a_j)$ then $a_j \in C_1$
Else if $U_2(a_j) > U_{-2}(a_j)$ then $a_j \in C_2$
\vdots
Else if $U_q-1(a_j) > U_{-(q-1)}(a_j)$ then $a_j \in C_{q-1}$
Else $a_j \in C_q$

Except for the hierarchical classification framework, the MHDIS method has another special feature that distinguishes it from other MCDA sorting methods as well as from other linear programming classification approaches (Stam, 1997). This involves the optimization framework used to develop the optimal sorting model (additive utility functions). In particular, during model development in the MHDIS method, three mathematical programming problems are solved at each stage $k$ of the hierarchical discrimination process ($k = 1,2,\ldots,q-1$). In particular, two linear and one mixed-integer programming problems are solved to estimate the ‘optimal’ pair of utility functions, where the term ‘optimal’ refers both to the total number of misclassifications as well as to the clarity of the distinction between the groups. Initially, a linear programming problem (LP1) is solved to minimize the magnitude of the classification errors (in distance terms). Then, a mixed-integer programming problem (MIP) is solved to minimize the total number of misclassifications among the misclassifications that occur after the solution of LP1, while retaining the correct classifications. Finally, a second linear programming problem is solved to maximize the clarity of the classification obtained from the solutions of LP1 and MIP. A detailed description of the model optimization process in the MHDIS method can be found in Zopounidis and Doumpos (2000a).

A.9. Decision rule models: the rough set theory

Pawlak (1982) introduced the rough set theory as a tool to describe dependencies among attributes,\(^\dagger\) to evaluate the significance of attributes and to deal with inconsistent data. Generally, the rough set approach is a very useful tool in the study of sorting and classification problems, regarding the assignment of a set of alternatives into pre-specified groups. Recently, however, there have been several advances in this field to allow the application of the rough set theory to choice and ranking problems as well (Greco et al., 1997, 2001). The rough set philosophy is founded on the assumption that with every alternative some information (data, knowledge) is associated. This information involves two types of attributes: condition and decision attributes. Condition attributes are those used to describe the characteristics of the alternatives (e.g. criteria), whereas the decision attributes define a partition of the alternatives into groups. Alternatives that have the same description in terms of the condition attributes are considered to be indiscernible. The indiscernibility relation constitutes the main mathematical basis of the rough set theory. Any set of all indiscernible alternatives is called an elementary set and forms a basic granule of knowledge about the universe. Any set of alternatives being a union of some elementary sets is referred to as crisp (precise) otherwise as rough (imprecise, vague). A rough set can be approximated by a pair of crisp sets, called the lower and the upper

\(^\dagger\) The traditional rough set theory refers to attributes instead of criteria. The attribute concept is similar to criterion concept. However, an attribute does not embody the preferential information that a criterion provides; it simply describes the alternatives.
approximation. The lower approximation includes the alternatives that certainly belong to the set and the upper approximation includes the alternatives that possibly belong to the set.

On the basis of these approximations, the first major capability that the rough set theory provides is to reduce the available information, so as to retain only what is absolutely necessary for the description and classification of the alternatives. This is achieved by discovering subsets of the attributes’ set, which provide the same quality of classification as the whole attributes’ set. Such subsets of attributes are called reducts. Generally, the reducts are more than one. In such a case the intersection of all reducts is called the core. The core is the collection of the most relevant attributes, which cannot be excluded from the analysis without reducing the quality of the obtained description (classification).

The subsequent steps of the analysis involve the development of a set of ‘IF . . . THEN . . .’ rules for the classification of the alternatives. The developed rules can be consistent if they include only one decision in their conclusion part, or approximate if their conclusion involves a disjunction of elementary decisions. Approximate rules are consequences of an approximate description of decision classes in terms of blocks of alternatives (granules) indiscernible by condition attributes. Such a situation indicates that using the available knowledge, one is unable to decide whether some alternatives belong to a given group (decision class) or not.

This traditional framework of the rough set theory, has been recently extended towards the development of a new preference modelling framework within the MCDA field (Greco et al., 1999, 2000). The main novelty of the recently developed rough set approach concerns the possibility of handling criteria, i.e. attributes with preference ordered domains, and preference ordered groups. Within this context the rough approximations of groups are defined according to the dominance relation, instead of the indiscernibility relation used in the basic rough sets approach. The decision rules derived from these approximations constitute a preference model.

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