Co-adaptive Strategies for Sequential Bargaining Problems with Discount Factors and Outside Options

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***Abstract*— Bargaining is a fundamental activity in social life. Game-theoretic methodology has provided perfect solutions for certain abstract models. Even for a simple model, this method demands substantial human intelligent effort in order to solve game-theoretic equilibriums. The analytic complexity increases rapidly when more elements are included in the models. In our previous work, we have demonstrated how co-evolutionary algorithms are used to find approximations to game-theoretic equilibriums of bargaining models that consider bargaining costs only. In this paper, we study more complicated bargaining models, in which outside option is taken into account besides bargaining cost. Empirical studies demonstrate that evolutionary algorithms are efficient in finding near-perfect solutions. Experimental results reflect the compound effects of discount factors and outside options upon bargaining outcomes. We argue that evolutionary algorithm is a practical tool for generating reasonable good strategies for more realistic bargaining models.**

I. Introduction

Bargaining situations are ubiquitous in our social life, from political parties' coalition for election to negotiation between husband and wife about domestic affairs, from threats of nuclear war to reaching new international trade agreements. Generally speaking, "bargaining" is a simple form of negotiation, mainly involving quantitative issues, such as price and quantity of products. Negotiation may cover many complex issues and terms, including deliverable time, payment methods, quality of service, compound goods and services, etc. A theory studies the essentials of bargaining situations is bargaining theory. It is one of the subbranches of Game theory. Nash bargaining model [1] is a fundamental and abstract one. Researchers in bargaining theory follow the general framework of Nash bargaining model, adding more realistic components into it to study more practical situations. For examples, researchers specify bargaining procedures, consider bargaining cost, risk of breakdown and outside and/or inside options and investigate information completeness.

This study extends our pervious work on simpler bargaining models, where only one bargaining factor, discount factor determines the bargaining powers. In this paper, we examine bargaining problems in the scope of sequential bargaining procedure, infinite-horizon extensive-form, complete and perfect information and having two bargaining factors that determine the bargaining powers, namely discount factors and outside options. The bargaining procedure goes in a sequential manner [2]. Infinite-horizon implies that the bargaining can potentially continue forever [3]. In a complete information game, players have complete information on

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each other's preference [3] [4]. Perfect information assumes that the player with the move knows the history of the game [3]. Discount factors measure the bargaining cost on time scale [5]. The existence of outside options potentially threats bargaining partners with withdrawal and ending up with nothing from the undergoing bargaining [6]. The idea of outside option can be illustrated by the following example bargaining scenario, which was originally given in [7]. Imagine University A and academic economist B is bargaining over B's annual salary. B has been offered a job by another institution with a fixed and nonnegotiable salary w_B. w_B is B's outside option. To make this problem simpler, let's assume that University A does not have an outside option (of replacing B) at the point of bargaining. The bargaining case between University A and academic economist B will exemplify how the existing outside options affect the bargaining outcome throughout this paper.

Typically, game theoretic research focuses on one or two determinants at one time and examines how these one or two determinants determine bargaining outcomes, outcomes' efficiency and stationarity. It is rare to examine many determinants (beyond three) in one theoretic model. It is probably due to the high level of complexity of models that have multiple determinants. Whilst the bargaining models increase the number of influential factors that affect the bargaining outcomes, the complexity becomes out of reach of human's ability of mathematic reasoning.

This motivates us to consider a substitutive method. Evolutionary algorithms are the methodology chosen here. Their applications show that they are able to deal with problems which are impractically treated by traditional methods, for example mathematic proofs. Evolutionary algorithms are especially suitable for problems which are non-linear, having large search space (for instance NP hard problems), multi-dimensional and dynamic problems. Assuming readers of this paper have sufficient background in evolutionary algorithms, we leave the introduction of evolutionary algorithms to [8], [9] and [10].

Our work simulates artificial players who are equipped with basic adaptive learning ability, a form of bounded rationality. Such ability is far from the perfect rationality assumed by the game-theoretic analytic method. These artificial players learn how to efficiently play outside option bargaining games through training experiences under an evolutionary algorithm scheme. Although the elements provided to the artificial learners are very primitive: arithmetic functions and variables, such learners' behaviors after sufficient trial-and-error training are very close to game-theoretic solutions and exhibit efficiency and stationarity.

In the remainder of this paper, first of all outside option bargaining problems and their game-theoretic solutions will be introduced in Section II. Experimental studies by evolutionary computation will be given in Section III, followed by experimental observations in Section IV. Conclusions and discussions are in Section V.

II. Outside Option Bargaining Models

A. Related research

Muthoo [7] mathematically models and analyzes the outside option situations in Rubinstein's alternating-offers bargaining scenario and investigates the relationship of their Subgame Perfect Equilibrium (SPE) with Nash bargaining product.

Binmore [6] models and solves a two-player and one-cake problem with an "outside option" pair by applying an asymmetric Nash bargaining solution that is the outcome of the axiomatic approach (one of game-theoretic approaches). In addition, the above solution is supported by a version of Rubinstein bargaining model with an outside-option pair. Further studies on this are fully investigated in [11] by both the strategic approach (one of game-theoretic approaches) and a human-subject experiment.

Binmore et. al [12] examine the Nash Demand game with outside options. An experimental study on this game shows that subjects make inefficient outcomes are common phenomenon. When the second player's outside option is sufficiently large, the mutual benefit for the bargaining often remains unexploited. Through experimental studies on demand and ultimatum games, Kahn and Murnighan [13] find that experimental data, as a whole, argue against game-theoretic quantity predictions in terms of opening demands, bargaining outcomes and efficiency. Such experimental studies are human-entry based.

B. Sequential Bargaining Problems with Outside Options

On the basis of the alternating-offer bargaining framework [5], we have studied a complete information model [14] and its four incomplete information variants [15]. In these problems, when a player is given an offer, he has only two choices: (1) acceptance thus ending the bargain with an agreement; or (2) rejecting the offer and making a counter offer after a time interval. We will integrate outside options into the above bargaining problem. In the presence of an outside option, a player has one more choice besides the two mentioned: he can choose to (3) end the bargain by taking his outside option, as illustrated in Figure 1. One player has no more than one outside option. If both players have outside options larger than 0, such a situation is called *two-sided outside option*. If one player has no outside option (or having an outside option equals 0) and another has an outside option larger than 0, it is called *one-sided outside option*. We treat one-sided outside option situations as special cases of two-sided outside option bargaining problems and as boundary checks of the values of outside options in experiments.

In a simple version of an outside option model, there are two players bargaining over a partition of a cake with its size 1. Two players are indexed by subscript i or j, $i, j \in \{1, 2\}$. All game related information are public. Their bargaining costs over time are measured by discount factors δ_1 and δ_2 respectively. Player 1 has his outside option $w_1 \in [0, 1)$ and the second player 2 has an outside option $w_2 \in [0, 1)$. If neither player has an outside options: $w_i = 0$ and $w_j =$

0, the bargaining model becomes the bargaining model in which discount factors are the sole determinant on bargaining powers [5]. In this work, we consider cases where at least one player has an outside option: $w_i > 0 \cup w_j > 0$.

To ensure that the bargaining is worth continuing and that no player prefers withdrawal from bargaining, the conditions $0 \leq w_1 < 1$, $0 \leq w_2 < 1$ and $0 < w_1 + w_2 < 1$ must be satisfied. It is necessary to investigate one-sided outside option cases as boundary checks. We include these special cases where only one player has outside option: $w_i = 0 \cap 0 < w_j < 1$, where $i \neq j$.

If $w_1 = 1$, or $w_2 = 1$ or $w_1 + w_2 \geq 1$, it is not mutually beneficial for dividing the cake, from the viewpoint of game-theoretic analysis. These conditions will also be examined in this study to see whether evolution algorithm is able to identify over-strong threats from outside option and to explore reasonable solutions.

When $w_i > 1$, an outside option is larger than the size of cake, there is meaningless to even start the bargaining because player i will take his outside option anyway. In short, this study will examine the first two situations in Table I, in which at least one player has an outside option larger than 0 and no outside option is larger than the size of cake. We furthermore, categorize the outside option, according to their values, as shown in Table II and III.

The outside option(s) stands statically since the bargaining starts and until the game ends. When an offer $x_i \in (0, 1)$ is accepted at the time t (t is an non-negative integer), player

TABLE I
BARGAINING PROBLEMS

Values of Outside Options	*Outside Option*	*Solutions*
$w_1 > 0$ AND $w_2 > 0$	Two-sided	Table III
($w_1 = 0$ AND $w_2 > 0$) OR ($w_1 > 0$ AND $w_2 = 0$)	One-sided	Table III
$w_1 = 0$ AND $w_2 = 0$	No	*Equations 1-2*

i receives a payoff $p_i = x_i \delta_i^t$ and the other player gets $(1 - x_i)\delta_j^t$. If one player i opts out the bargaining at the time t and takes his outside option, he receives a payoff $w_i \delta_i^t$ and another player j will take her outside option and gets a payoff $w_j \delta_j^t$. If they both perpetually disagree and do not take their outside options, then both players obtain 0. Note that in the models studied here, the outside option(s) is discounting over time at the same rate as the cake. The rate is the player's discount factor. This ensures that players make decisions under time pressure. [1]

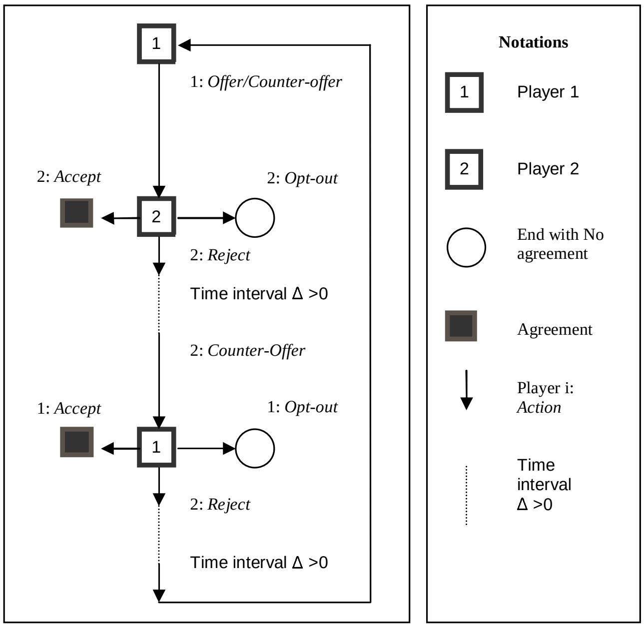

Fig. 1. Outside Option Bargaining Scenario

[1]In game theory literature, outside options in some bargaining models have no connection to time preferences.

TABLE II

OUTSIDE OPTION BARGAINING PROBLEMS OVERVIEW

Category Name	*Outside Option*
Ineffective Threats	C1
Effective Threats	C2
Over-strong Threats	C3

C. Game-theoretic Solutions: Subgame Perfect Equilibrium

The unique *Subgame Perfect Equilibrium* (SPE) solution (x_1^*, x_2^*) of outside options is stated in the Table III where,

$$\mu_1 = \frac{1-\delta_2}{1-\delta_1\delta_2} \quad (1)$$

$$\mu_2 = \frac{1-\delta_1}{1-\delta_1\delta_2} \quad (2)$$

Note that μ_1 and μ_2 are Subgame Perfect Equilibrium for the corresponding sequential bargaining problem without outside options. x_1^* and x_2^* are players' strategies in the state of Subgame Perfect Equilibrium. Whenever one player makes an offer or a counteroffer, he only asks for the SPE share x_i (*Stationarity*). The shares from an agreement in SPE depend on who is the first player. We assume that player 1 is the player who makes the first move. His offer decided by his SPE strategy x_1^* will be accepted by player 2 immediately (*No delay*). Player 2 will get a share $1-x_1^*$ from this agreement, except $\#e$ in Table III. So the shares for player 1 and 2 from the SPE agreement will be $(x_1^*, 1-x_1^*)$. All SPE agreements should be reached at the bargaining time $t=0$. Therefore, player 1 and 2 obtain payoff x_1^* and $1-x_1^*$ respectively. For Condition $\#e$, theoretically players take their outside options straightway because for at least one player, his outside option is more beneficial than any possible payoff from a bargaining agreement.

TABLE III

SPE UNDER 5 DIFFERENT CONDITIONS FOR OUTSIDE OPTION BARGAINING PROBLEMS. IN OUR EXPERIMENTS, PLAYER 1 MAKES THE FIRST OFFER. THE SHARES IN A SPE AGREEMENT IS $(x_1^*, 1-x_1^*)$ UNDER THE CONDITIONS OF a, b, c AND d. UNDER THE CONDITION e, $x_1^* = w_1$ AND $x_2^* = w_2$

#	x_1^*	*Conditions (AND)*		C.
		I	II	
a	μ_1	$w_1 \le \delta_1\mu_1$	$w_2 \le \delta_2\mu_2$	C1
b	$1-w_2$	$w_1 \le \delta_1(1-w_2)$	$w_2 > \delta_2\mu_2$	C2
c	$\delta_2 w_1 + (1-\delta_2)$	$w_1 > \delta_1\mu_1$	$w_2 \le \delta_2(1-w_1)$	C2
d	$1-w_2$	$w_1 > \delta_1(1-w_2)$	$w_2 > \delta_2(1-w_1)$	C2
e	w_1	$w_1 + w_2 > 1$	-	C3

D. Game-theoretic Assumptions: Economic Man

The problems above have been solved by means of game-theoretic methods. The solution implies the assumption of perfect rationality. This assumption entails that each player takes everything into consideration including the opponent's reasoning, which is recursive.

Simon [16] firstly questions the perfect rationality assumed on participants of games in traditional economic theory. "Economic man" is described as having sufficient information relevant to the game; stable preferences and enough computational resources. Instead human behaviors are bounded rational. The application of evolutionary algorithms allows the adoption of boundedly rational assumptions on players' behaviors. A player then is modeled as an "administrative man" who "satisfices - looks for a course of action that is satisfactory or 'good enough.' " [17].

E. Computational Complexity of Bargaining Problems

Outside Option bargaining problems are intractable if an exhaustive search method is in use. Take a two-sided outside option problem as an example, one player can make potentially infinitely possible offers. Moreover, from the definition of infinite-horizon, the bargaining procedure can last forever. To simplify the problem, we limit that a bargaining only lasts 10 time intervals and the divisions of the cake have only 10^{-2} precision. At one time, the offering or counter-offering player has 100 options (the cake size $\pi = 1$ is divided by 100) while the another one has three options: acceptance, rejection or opting out. So at a particular time, there are $100 \times 3 = 300$ possible outcomes. For 10 time intervals, there are $300^{10} \approx 6 \times 10^{24}$ possibilities. If a machine tests possibilities at the rate of one possibility per nano-second (10^{-9} second), it requires 6×10^{15} seconds, or more than 180 millions years to test all possibilities for one single bargaining problem. When more determinants taken into consideration, the complexity of such a problem goes beyond the time constraint of an exhaustive search we can afford or requires excessive human efforts. So an alterative method is heavily in demand. Evolutionary algorithm as a heuristic search method is probably able to generate reasonable good solutions for such problems within reasonable time and computational expenses. More specifically the replacement approach we have chosen is a co-evolutionary algorithm. It will be elaborated in Subsection III-A.

III. Experiments

We have established a co-evolutionary system to study complete information bargaining problems under time preferences [14] and four incomplete information bargaining problems [15]. This system will be reused with slight modifications to meet the description of outside option bargaining problems. This system is named as *Constraint driven Co-evolution Genetic Programming*, or *CCGP*.

A. Evolutionary Algorithm

Evolutionary algorithm is a biologically, specially genetically, motivated information processing system. It follows the general principles of natural evolution, the fittest survive under natural selection. New populations emerge after recombination and variation.

As having discussed in [14] [15], we identify two special features of the two-player bargaining problems: players interact with each other; and players learn through trial-and-error experiences that train adaptive players to learn how to make profitable decisions. *Co-evolution* is an ideal simulation to address these features. In biology, co-evolution is reciprocal evolutionary changes in interacting species. Thus, we use a co-evolutionary algorithm with two populations to simulate the two players' co-evolving relationship. The structure of the two-population co-evolution system is illustrated in Figure 2.

B. Experimental Design

The established co-evolutionary system is implemented by means of Genetic Programming (GP) [9]. The major modifications on the co-evolutionary system to satisfy the requirements of the introducing outside options into the

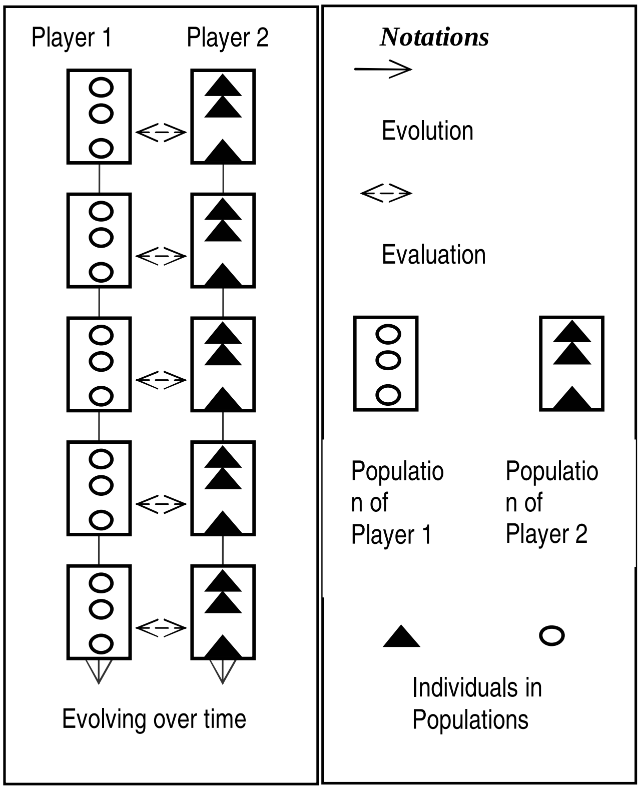

Fig. 2. Two-population Co-evolution

problems are in three aspects: (i) Outside option(s) are one of two determinants on bargaining outcomes. As the information about outside options are defined as public, their values are added into the terminal sets of genetic programmes of both populations. (ii) The existence of an outside option provides an alternative choice for a player when he makes decisions. The bargaining procedure consequently changes to allow a player to secede after refusing an offer and to take up his outside option; (iii) the game fitness of a genetic program becomes $x_i\delta_i^t$ if an agreement settled at t or $w_i\delta_i^t$ if player i takes up his outside option w_i at t.

C. Genetic Program

Each player has a set of candidate solutions, a population. A candidate solution in a population is a genetic program, g_i for player i, $i \in \{1, 2\}$. A genetic program's terminal set consists of variables and constants as nodes and its functional set consists of functions and operations which take members of the terminal set as input(s). Players both have complete information relevant to the bargaining game, therefore their information set is $\{\delta_1, \delta_2, w_1, w_2\}$. Added the size of cake 1 and the -1 to change the sign, the terminal set for g_i is $\{\delta_1, \delta_2, w_1, w_2, 1, -1\}$. As far as the function set concerns, we choose arithmetic functions $\{+, -, \times, \div\}$, because they are fundamental and the SPEs are expressed arithmetically so arithmetic functions should be sufficient for the problem. GP parameters and their values are displayed in Table IV.

TABLE IV
SUMMARY OF THE GENETIC PROGRAMMING PARAMETERS

GP parameters	*Values*
Terminal set	$\{\delta_1, \delta_2, w_1, w_2, 1, -1\}$
Functional set	$\{+, -, \times, \div\}$ ($\div$ is Protected)
Population Size	100
Number of Generations	300
Initial Max Depth	5
Maximum nodes of a GP program	50
Initialization Method	Grow
Selection Method	3-member Tournament
Crossover rate	(0, 0.1)
Mutation Method	Sub-tree Mutation
Mutation rate	(0.01, 0.3)

D. Bargaining Scenario

A genetic program g_i's corresponding time-dependent bidding function is defined as $b(g_i) = g_i \times (1-r_i)^t$, where r_i is the discount rate, $\delta_i \equiv exp(-r_i)$. $b(g_i)$ ensures that player i bids in decreasing shares. The nonnegative integer t is the bargaining time.

A strategy determines what action (acceptance, counteroffer or ending bargaining) it takes at time t upon an offer or a counter offer. The strategy s_i responses to an offer $(1-x_j, x_j)$ at time t:

$$s(g_i) = \begin{cases} \textit{Accept} : x_i = 1 - x_j \\ \quad \text{if } (1-x_j)\delta_i^t \geq MAX(b(g_i)_{(t+1)}\delta_i^{t+1}, w_i\delta_i^t) \\ \\ \textit{Opt-out} \ : x_i = w_i \\ \quad \text{if } w_i\delta_i^t > MAX(b(g_i)_{(t+1)}\delta_i^{t+1}, (1-x_j)\delta_i^t) \\ \\ \textit{Counteroffer at } (t+1) : x_i = b(g_i)_{(t+1)} \\ \quad \text{if } b(g_i)_{(t+1)}\delta_i^{t+1} > MAX((1-x_j)\delta_i^t, w_i\delta_i^t) \end{cases} \quad (3)$$

$s(g_i)$ expresses a player's actions to max his payoff. A player chooses the most beneficial one among three possible options: accepting the current offer immediately; taking his outside option now; or waiting for counter offer. If accepting the current offer brings no less payoff than either counter-offering after one time interval or taking his outside option now, the player accepts the offer; if taking outside option returns higher payoff than either counter-offering after one time interval or accepting the current offer, the player takes his outside option; if counter-offering after one time interval will probably bring higher payoff than either accepting now or taking his outside option, the player counter-offers.

E. Fitness Evaluation

g_i's payoff from an agreement with g_j of another player j's population, or from taking his outside option, is notated as $u_{s(g_i)\rightarrow s(g_j)}$. If both players agree with a division of the cake as $(x_i, x_j) = (x_i, (1-x_i))$ at bargaining time t, then g_i gets $x_i \times \delta_i^t$; if one of players decides to take his outside option, g_i gets $w_i \times \delta_i^t$.

$$u_{s(g_i)\rightarrow s(g_j)} = \begin{cases} x_i \times \delta_i^t \\ \quad \text{if } makes \ agreement \\ w_i \times \delta_i^t \\ \quad \text{if } takes \ w_i \end{cases} \quad (4)$$

The game fitness of g_i, $GF(s(g_i))$ is the average payoff that $s(g_i)$'s gains from agreements or from taking his outside option with individuals in the co-evolving population J which is a set of m individuals, $j \in J$. The integer m is an experimental parameter.

$$GF(s(g_i)) = \frac{\sum_{j\in J} u_{s(g_i)\rightarrow s(g_j)}}{m} \quad (5)$$

These bargaining problems have a hard constraint: $x_i \in (0,1]$, so any offer, counter-offer or a share of an agreement should not be larger than the size of the cake. If this constraint is satisfied, an extra values 3 is added on the top of its game fitness. Otherwise, a penalty $h(g_i)$ is applied. This encourages feasible individuals to propagate in the new population. This is an application of Incentive method, a constraint handling technique in evolutionary algorithm. The Incentive method with case studies have been reported in [18].

g_i's overall fitness $F(g_i)$ is defined as:

$$F(g_i) = \begin{cases} GF(s(g_i)) + 3 \\ \quad \text{if } g_i \in (0,1] \\ h(g_i) \\ \quad \text{if } g_i \notin (0,1] \end{cases} \quad (6)$$

where $0 \leq h(g_i) \leq 2$.

F. Examinations

As there are several categories and conditions, it is necessary to divide experimental tests in groups. We will test three categories of situations that are divided by the values of outside options and by their relationship with discount factors as in Table III.

1) *Category 1: Ineffective Threats*: cases where $w_1 \leq \delta_1 \mu_1$ and $w_2 \leq \delta_2 \mu_2$, condition a in the Table III, including one-sided outside option tests.
2) *Category 2: Effective Threats*: game parameters under the three conditions $\#b, c$ and $\#d$ in Table III, including one-sided outside option testing cases.
3) *Category 3: Over-Strong Threats*: condition $\#e$ in Table III, including cases $w_1 + w_2 > 1 \cap 0 < w_1 < 1 \cap 0 < w_2 < 1$, and boundary checks of one-sided cases $w_1 = 1 \cap 0 < w_2 < 1$; $0 < w_1 < 1 \cap w_2 = 1$.

Values of game parameters are chosen as follows: δ_1 and $\delta_2 \in \{0.1, 0.5, 0.9\}$, representing small, middle and large discount factors. $w_1 \in \{0, 0.01, 0.03, 0.1, 0.2, 0.4, 0.5, 0.7, 1\}$ and $w_2 \in \{0, 0.01, 0.02, 0.03, 0.05, 0.1, 0.2, 0.4, 0.5, 0.7, 1\}$. These values are not fully combined. Results will be grouped by game parameters' relationship according to conditions defined in Table III.

For each game setting, we execute 100 trials, starting with different random seeds. Generally speaking, to sample 100 data ensures the statistical sufficiency. We will record (1) the observed average shares $\bar{x_1}$ and $\bar{x_2}$ of 100 trials. If $\bar{x_1} + \bar{x_2} = 1$, only $\bar{x_1}$ will be listed. For cases where $\bar{x_1} + \bar{x_2} \neq 1$, as expected for Category $C3$, both $\bar{x_1}$ and $\bar{x_2}$ will be given; (2) the average time $\bar{t}$ for reaching agreements in experiments and (3) the deviations σ of shares x_1s and x_2s of 100 trials will be used to examine the evolutionary stability of solutions.

TABLE V
T-TEST RESULTS UNDER 5 DIFFERENT CONDITIONS FOR OUTSIDE OPTION BARGAINING PROBLEMS. 95% CONFIDENCE LEVEL

#	*t Statistical value*	*t Critical two-tail*
a	0.4592	2.0423
b	0.9112	2.0049
c	0.4020	2.0154
d	0.4704	2.0739
e	-1.6506	1.9769

IV. OBSERVATIONS

In order to compare experimental results with game-theoretic analytic solutions, we measure their differences with respect to divisions in agreements and bargaining time. The payoff and efficiency of bargaining settlements can be worked out from divisions of agreements and bargaining time [2]. In addition, to demonstrate the efficiency of evolutionary algorithms, the computational resources required to deal with these problems will also be examined.

A. Shares in agreements

Experiments are split into three categories with five conditions as in Table III. We make t-tests on the hypothesis $x_1^* - \bar{x_1} = 0$ for five conditions. The hypotheses under these five conditions are all accepted with 95% confidence, see Table V. They are the statistical evidences that the data sets compared, namely x_1^* and $\bar{x_1}$ do not have statistically significant difference.

We will analyze the compound impacts of two determinants: discount factors and outside options on bargaining powers on the basis of experimental results.

[2] It is too large to insert the raw experimental results into this paper. They are available on line http://privatewww.essex.ac.uk/∼njin/CEC06.htm

1) *Category 1: Ineffective Threats* $w_1 \leq \delta_1\mu_1$ and $w_2 \leq \delta_2\mu_2$: condition a in the Table III. Theoretically, the values of outside options are too small to make any credible threats on the outcomes at all. Bargaining agreements are better off than players' outside options. Thus no one considers his outside option. Bargaining will continue as if there is no outside options. Theoretic results for this circumstance is the same as the SPE solutions for the sequential bargaining problem that has no outside option, i.e. $(\mu_1, 1-\mu_1)$. Let us return to our bargaining example of University A and economist B. Suppose B's current salary is $\pounds 50,000$ and University A obtains $\pounds 50,000$ (50% of the mutual benefit) from having B working at University A. The size of cake is $\pounds 100,000$. If B's alternative job offer provides a salary $\pounds 49,999$ (his outside option w_B). B's threat to quit is incredible because University A will just ignore this threat. Experimental results support the game-theoretic analysis. In our experimental data [19], for example, Table 1: $\#2$ $(\delta_1, \delta_2, w_1, w_2) = (0.1, 0.9, 0, 0.7)$ where the player 2' outside option seems a strong one $w_2 = 0.7$. The observed $\bar{x_1}$ is 0.1104 (SPE x_1^* 0.1099). For the bargaining situation having no outside option $(\delta_1, \delta_2, w_1, w_2) = (0.1, 0.9, 0, 0)$, the SPE x_1^* is 0.1099. We can see that in this case, the bargaining outcome does not matter whether the outside option $w_2 = 0.7$ exists. Compared with bargaining power from the relative magnitude of respective bargaining costs (δ_1, δ_2), the $w_2 = 0.7$ is too small to increase player 2' bargaining power.

2) *Category 2: Effective Threats* The conditions b, c and d in Table III; in such circumstances, theoretically players' outside option(s) are effective cause(es) to increase one of players' bargaining power while both players still prefer dividing the cake to choosing their outside options. Players do not take up their outside option and remain at negotiation table, but the presence of outside option(s) influence the partition of cake. Suppose this time, economist B receives a job offer $w_B = \pounds 55,000$. B's threat to quit is now credible and he increases his bargaining power due to the value of w_B. University A compromises to the extent that A only needs to increase B's salary to the exact value of w_B in order to keep having B working here. University A does not need to give $\pounds 1$ extra more than $w_B = \pounds 55,000$. "Credible threats and credible promises matter." [20]. In the experimental data [19], for example, Table 2: $\#1$ $(\delta_1, \delta_2, w_1, w_2) = (0.1, 0.1, 0, 0.7)$. The observed $\bar{x_1}$ is 0.2663. For the bargaining situation having no outside option $(\delta_1, \delta_2, w_1, w_2) = (0.1, 0.1, 0, 0)$, the SPE x_1^* is 0.9091 (so $x_2^* = 0.0909$). In this case, the existence of the outside option $w_2 = 0.7$ makes difference on the bargaining outcome. w_2 significantly increases player 2's bargaining power from a 0.0909 slide of the cake to an offer from player 1 who instead offers 0.7337 to player 2 (in SPE $x_2^* = 0.7$), the same value as player

2's outside option 0.7. Player 1's bargaining power decreases accordingly.

3) *Category 3: Over-Strong Threats* $0 < w_1 < 1$, $0 < w_2 < 1$, and $w_1 + w_2 > 1$: condition $\#e$ in the Table III. From the game-theoretic analysis, it is expected that at least one player prefers his outside option to bargaining, so in equilibrium both players take their outside options. If economist B is offered a job with the salary $w_B = \pounds 100,000$, it is intuitive that B will walk away from the negotiation table. So does University A. If increasing B's salary to his outside option $\pounds 100,000$, University A obtains nothing from keeping B, because there is indifference between to have B working here and to have nobody working for his position at all. It is a situation where there is no mutual benefit of bargaining as the cake disappears. Experimental results on the partitions of a cake agree with game theoretic predictions. In the experimental data [19], for example, Table 5: $\#2$ $(\delta_1, \delta_2, w_1, w_2) = (0.1, 0.1, 0, 1)$, player 2's outside option is the same size of the cake. The observed $\bar{x_1}$ is 0 and $\bar{x_2}$ is 1. Obviously, player 2 takes his outside option, no point to carry out bargaining.

To analyze the resulting bargaining time and stationarity in experiments, we separate results of the game settings by "$\#a, b, c$ and d" and "$\#e$". The reason is that in theory, bargaining under conditions "$\#a, b, c$ and d" end up with bargaining agreements and situations under $\#e$ results in taking up outside options.

TABLE VI
AVERAGE BARGAINING TIME $\bar{t}$ UNDER 5 DIFFERENT CONDITIONS FOR OUTSIDE OPTION BARGAINING PROBLEMS.

range of $\bar{t}$	Percentage over game settings under Conditions $\#a, b, c$ and d	under $\#e$
$\bar{t} = 0$	47%	67%
$0 < \bar{t} < 0.05$	20%	17%
$0.05 < \bar{t} < 0.10$	16%	8%
$0.10 < \bar{t} < 0.50$	14%	8%
$0.50 < \bar{t} < 4.00$	3%	0%
Average value of $\bar{t}$	of Conditions $\#a, b, c$ and d	$\#e$
$\bar{t} =$	0.1237	0.0229

B. Bargaining Time

The bargaining time t^* for reaching the SPE for all game settings are 0. In theory, for all conditions $\#a, b, c, d$ and $\#e$, the equilibrium solutions imply $t = 0$. As shown in the payoff function $u_i = x_i \times \delta_i^t$, the payoff deteriorates exponentially while t increases. t is the indicator of the efficiency of an agreement. An agreement stroke at $t = 0$ is the most efficient one. Any $t > 0$ suggests inefficiency as players pay the cost for delays. Experimental results of average bargaining time for reaching agreements are stated in Table VI. Generally speaking the experimental results of $\bar{t}$s are very small. It is obvious that when threats from outside options are over strong, (condition $\#e$), players learn to take their outside option almost immediately. When players prefer to bargaining they spend longer time to reach agreements. In Subsection II-A, we have surveyed some related studies in the field of experimental economics on outside option bargaining problems. These studies have found that outcomes of human decisions are inefficient in general. Compared with decisions made by human subjects, those by artificial players

TABLE VII

DEVIATION σ OF $\bar{x_1}$S UNDER CONDITIONS $\#a, b, c, d$ AND $\#e$ OF OUTSIDE OPTION BARGAINING PROBLEMS.

σ	Percentage of tests under Condition $\#a, b, c$ and d	under $\#e$
Average	0.0209	0.0139
Maximum	0.0430	0.0759

demonstrate higher efficiency. As inefficiency observed in our experiments is so small that it can be considered as the consequence of the stochastic property of evolutionary algorithm.

C. Stationarity

We measure the stationarity of strategies by means of examining the deviation σ of $\bar{x_1}$s in their final population. Experimental results are shown in Table VII. Both the average and maximum values of deviations are very small, showing that no player wants to withdraw from such agreements under the conditions $\#a, b, c, d$ or to take choices other than outside options under the condition $\#e$.

D. Computational Resources

A Linux machine with an $athlon2400$ processor runs about 1 hour to test a set of game parameters $\{\delta_1, \delta_2, w_1, w_2\}$ (100 runs). We only spent a few days to run experiments for 115 sets of game parameters. Compared with either an exhaustive search or game-theoretic method, the artificial simulation approach is much more efficient in terms of time and computational resources.

V. CONCLUSIONS AND DISCUSSION

In our previous work, we have studied bargaining models which have one determinant on bargaining outcomes: discount factor. We have developed a Constraint based Co-evolution Genetic Programming, CCGP system to tackle the problem. The system has generated strategies approximate game-theoretic solutions.

In this work, we add another determinant: outside option into bargaining models, beside discount factor. Having more determinants, bargaining problems become more complex and therefor more difficult to be solved by mathematic based game-theoretic method. We aimed to investigate whether the CCGP system is able to generate reasonable good strategies for the more complex bargaining problems: bargaining problems with both discount factors and outside options. CCGP system is reused with slight modifications for these problems.

From experimental results, the mutual benefits (the cakes) are partitioned in a way that approximate the Subgame Perfect Equilibrium. The compound effects of discount factors and outside options on bargaining outcomes demonstrated in experiments support the game-theoretic analysis. The average bargaining time is very small, meaning that the agreements in experiments are of nearly perfect efficiency. Additionally, observed players' behaviors in making agreements show high stability. The computational resources for tackling these problems are relative low. The experimental results and observations enhance our assertion that evolution algorithm, particularly the CCGP system is capable of finding out nearly perfect solutions within manageable computational resources and time.

Real bargaining situations potentially have infinite determinants that affect their outcomes. Therefore, it is always difficult to predict bargaining outcomes and find equilibriums. Artificial simulations, may help release such analytic burden. It will be a major advance if we engineer the improved CCGP system to simulate multi-determinants bargaining events.

We are hoping to feed in more determinants into the CCGP simulator at one time and expecting CCGP to generate hints for possible outcomes and recommend helpful strategies. It should be able to cope with many important factors, although not all of factors from a real situation. Please note that a CCGP simulation is not designed to "mathematically solve" realistic bargaining problems. Whilst, the quality of solutions, the efficiency on resources and CCGP's reusability are attractive points for considering using such an artificial simulation.

In summary, from studying these bargaining models, we are gaining more confidence in using evolutionary algorithms and the CCGP system to investigate more realistic bargaining situations whose game-theoretic solutions are unavailable yet. Strategies derived from CCGP are probably approximate unknown game-theoretic equilibriums.

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