Equilibrium Selection by Co-evolution for Bargaining Problems under Incomplete Information about Time Preferences

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Abstract- The main purpose of this work is to measure the effect of bargaining players' information completeness on agreements in evolutionary environments. We apply Co-evolutionary Algorithms to solve four incomplete information bargaining problems. Empirical analyses indicate that without complete information of player(s)'s preferences, co-evolving populations are still able to select equilibriums which are Pareto-efficient and stationary. This property of the co-evolutionary algorithm supports its future applications on complex dynamic games.

1 Introduction

Most of complicated games are modeled in presence of incomplete information and/or imperfect information. The two-player bargaining problems we attempt to study are dynamic infinite-horizon games under incomplete information and perfect information. In an *incomplete information* game, at least one player is uncertain about another player's utility function. Such as Bridge and Poker, no player is supposed to know his opponents' utility functions. In a *perfect information* game, at each move, the player with the move knows the full history of the play of the game thus far [Gibbons 1992]. In board games, players observe the history of moves and the states. So do most bargaining problems where players experience what and when the other offers and counter-offers.

Popular approaches to study games include theoretically analytic method, behavior method, evolutionary game theory approach and artificial simulations. A considerable amount of research shows that artificial simulations by nature-inspired techniques can successful provide competitive solutions for many games.

Axelrod [Axelrod 1987] and Miller [Miller 1996] apply a Genetic Algorithm (GA) to study Iterated Prisoners' Dilemma under complete information: both players knows the payoff table; Koza [Koza 1992] employs Co-evolution in conjunction with Genetic Programming (GP) to disclose minimax strategies for a two-player finite-size games in extensive form under complete information. [Cliff and Miller1996] studies predator-prey games. [Jin & Tsang 2005] reports that a co-evolutionary algorithm generates results that approximate the analytic solution for a two-player sequential bargaining game under complete information setting.

[Billings et al. 2002]'s simulations of Porker, an incomplete-information game and modeling of the opponent involve the Neutral Network. Clemens and Reichmann examine evolutionary dynamics of agents in contribution games and subscription games under incomplete information. The outcomes of simulations conducted by an Evolutionary Algorithm (EA), support their theoretic analysis [Clemens & Reichmann 2003]. They did not execute simulations under correspondingly complete information setting to check whether the information completeness leads to any different solution.

The characteristic of incomplete information capture an essential aspect of real world bargaining. In few occasions players perceive full knowledge of their opponents. It is important to study how players' information state influences the bargaining outcomes and if by Evolutionary Algorithms approach whether a (co-)evolving population provides inefficient or disadvantageous solutions for a player who is not fully informed by the game related information.

To meet this end, we introduce four problems imposed by different inputs on the players' knowledge. To be comparable with results in [Jin & Tsang 2005], we restrict attention to the bargaining problems with the most minor modification on the basis of the complete information model [Rubinstein 1982]. The four problems depart from the complete information bargaining problem only in that players have incomplete information about the other's preference. The bargaining scenario, utility functions and the study approach remain unchanged.

We apply one of the Evolutionary Algorithms, Co-evolutionary Algorithms to solve these bargaining problems, assuming that (1) the performance of a player's strategy depends on the strategies employed by his opponent; (2) players learn other's behaviors through bargaining experiences; (3) players are trying to find out the best responses(s) to the other's strategies so as to maximize their own utility; (4) players have the same level of learning ability.

Experimental results and analysis indicate that evolutionary populations successful reach (one of) the most efficient and stationary equilibria. The efficiency means that agreements reached at the moment the game starts and thus there is no any bargaining cost incurred. The stationariness means that if both players are perfectly rational and have ability of conducting game-theoretic analysis, they will not unilaterally withdraw from this equilibrium.

The remainder of this paper will study four sequential bargaining problems. They are: both two players have no information about the other (NoInfo), only one player has information about the other (ASY), one player has inaccurate information of the other (INA), and one player has partial information about the other (ICRub85). Among them, the ICRub85 has game-theoretic solutions.

2 Static Incomplete Information Problems

We define two types of incomplete (missing) information in terms of problems:

1. *Static Incomplete Information (Missing Constant)*: the missing information is existing, fixed and effective in solutions. For example, in the NoInfo bargaining problem, the players 1 and 2 do not know the other's time preferences' value δ_2 and δ_1 respectively. Such values exist as constants in the player i' utility functions: $x_i\delta_i^t$ where $i \in \{1, 2\}$.
2. *Dynamic Incomplete Information (Missing Variable)*: the missing information is a variable which can change its values. For instance, in a bargaining problem, if the preference of a player is a variable, changing its value over bargaining time or evolutionary time, the problem are far more difficult to solve.

In this context, only will the static incomplete information be investigated. The dynamic incomplete information situations will leave unresolved and be briefly discussed in the section of Discussion.

3 Bargaining Problems

In a typical bargaining situation, participants have a common interest (often called 'a cake') but they conflict with how to divide the cake. So they go through bargaining procedure in order to make an agreement over the partition.

Five two-player bargaining problems will be described, varying in players' knowledge in the aspect of the other player's preference. Players' preferences are solely measured by their bargaining costs per time interval, termed as discount factors δ_1 and δ_2 for the player 1, and for the player 2 respectively. A player who has a higher discount factor spends less than a player who has a lower discount factor on one bargaining time interval. As a bargaining going, players' utilities are discounted over time, so both players have incentives to reach an agreement as soon as possible. A player is assumed to have correct information about his own δ_i. Here, δ_i is a real number, $\delta_i \in (0, 1)$ where $i \in \{1, 2\}$. For both players, their awareness of $\delta_i \in (0, 1)$ is implicit in all situations. The player 1 starts bargaining over the surplus (a cake) of the size 1 at the time $t = 0$, t being an non-negative integer. Two players make offers and counteroffer in a strictly alternating manner. Any offer will be either accepted immediately or rejected and counter-offered after one time interval. Once an offer is accepted, an agreement is reached with a share x_1 for the player 1 and a x_2 for the player 2, $x_1 + x_2 = 1$. If this agreement is reached at time t, their two players attain the utilities $u_1 = x_1\delta_1^t$ and $u_2 = x_2\delta_2^t$ respectively. The length of one bargaining period is one time interval Δ. To make things simpler, we assume $\Delta = 1$.

3.1 Complete-Information Bargaining Problems - CRub82

The classic complete-information bargaining model is named as Alternating-Offer bargaining problem or Rubinstein Bargaining Problem [Rubinstein 1982]. In this case, both players have correct information on the other's time preferences. Its *subgame perfect equilibrium* (SPE) of this problem is the unique point where "no delay" and "stationarity" properties hold. The SPE is

$$x_1^* = \frac{1 - \delta_2}{1 - \delta_1\delta_2} \tag{1}$$

and the player 2 will certainly accept it at the $t = 0$. Therefore, $u_1 = x_1^*$ and $u_2 = 1 - x_1^*$. There is no any bargaining costs spend. Proof and technical treatments are available in [Rubinstein 1982], [Muthoo 1999], and [Osborne & Rubinstein 1990].

3.2 Incomplete Information Bargaining Problems - ICRub85

On the basis of the complete-information alternating-offer bargaining problem aforementioned, Rubinstein [Rubinstein 1985] studies an alternating-offer bargaining situation with incomplete information. The player 1 only knows that 2's time preference δ_2 is either δ_w or δ_s whereas 2 knows 1's time preference δ_1. δ_s is strictly larger than δ_w, meaning that a player 2 with δ_s spends a less bargaining cost than a player 2 with δ_w for one time interval. The possibility of the player 1's initial belief on the player 2's type being δ_w is denoted by ω_0.

Analytic theorem proves that when ω_0 is high enough such that $\omega_0 > \omega^*$,

$$\omega^* = \frac{V_s - \delta_1^2 V_s}{1 - \delta_w + \delta_1 V_s(\delta_w - \delta_1)} \tag{2}$$

the only *Perfect Bayesian Equilibrium* (PBE) is that the player 1 offers x^{ω_0} for himself:

$$x^{\omega_0} = \frac{(1 - \delta_w)(1 - \delta_1^2(1 - \omega_0))}{1 - \delta_1^2(1 - \omega_0) - \delta_1\delta_w\omega_0} \tag{3}$$

and the player 2_w accepts it and the game over. 2_s rejects it and makes a counter-offer y^{ω_0} for himself:

$$y^{\omega_0} = x^{\omega_0}\delta_w \tag{4}$$

which will be accepted by the player 1. The PBE is $1 - y^{\omega_0}$ for the player 1 and the rest of the cake obtained by the player 2.

In the case of $\omega_0 < \omega^*$, the player 1 offers what he offers to a 2_s with V_s which will be accepted by both 2_w and 2_s. The PBE is V_s for the player 1 and $1 - V_s$ for the player 2.

$$V_s = \frac{1 - \delta_s}{1 - \delta_1\delta_s} \tag{5}$$

Bargaining Problems	*Player 1's knows*	*Player 2 knows*
CRub82	δ_1, δ_2	δ_1, δ_2
ICRub85	$\delta_1, \omega_0, \delta_w, \delta_s$	δ_1, δ_2
INA	$\delta_1, (\delta_2, \sigma)$	δ_1, δ_2
ASY	δ_1	δ_1, δ_2
NoInfo	δ_1	δ_2

Table 1: Two players' knowledge of the other's preference in the five bargaining problems. σ is the deviation of a Gaussian distribution.

If $\omega_0 = \omega^*$, there are more than one bargaining PBE are possible . From the Perfect Bayesian Equilibria above, it is obvious that the actual value of the $\delta_2 \in \{\delta_w, \delta_s\}$ theoretically, has no immediate effect on the player 1's first offer, but his initial belief ω_0 and the combination of δ_w and δ_s have.

3.3 Inaccurate Information - INA

This model interests us in that the player 1 is not sure about the exact value of 2's δ_2, but knows it is drawn from a Gaussian distribution with the mean of δ_2, see Table 1. The player 1 in this problem has less precise information on the player 2's preference than the player 1 in ICRub85.

[Ordover & Rubinstein 1983] analyzes a bargaining game that players learn others' specific characteristics (bargaining powers) which are drawn from a probability distribution that is unknown to others. The major differences of their work from the INA are in two key aspects (1) [Ordover & Rubinstein 1983] has several sellers and the buyer can switch sellers. Instead the INA is only designed for two players where both players' time preferences are prefixed, although one of them only knows the other's preference is from a distribution. (2) [Ordover & Rubinstein 1983]'s sellers and the buyer have correlated values on the indivisible object, namely their valuations to the object. The buyer wants to learn about the sellers' valuations as well as their bargaining powers [Fudenberg et al. 1985]. On the contrary, in INA the common interest (the cake) is public information. The only purpose of learning is to discover the other' bargaining power. [Grossman & Perry1985] also analyzes an incomplete information bargaining game on an indivisible object and the buyer's type is in a probability distribution.

3.4 Asymmetric information - ASY

[Fudenberg et al. 1983] solves a two-person two-period bargaining problem under incomplete information. A buyer and a seller bargain over an indivisible object. The valuation of the seller to the object is common knowledge but the buyer's valuation is private. They prove the existence of a unique Perfect Bayesian Equilibrium. We introduce another asymmetric information problem in the scope of bargaining over a divisible interest, infinite-horizon and only learning time preferences. The player 1 knows nothing about the value of δ_2. The player 2 knows the δ_1.

GP parameters	*Values*
Population Size	100
Number of Generations	300
Initial Max Depth	5
Initialization Method	Grow
Selection Method	3-member Tournament
Crossover rate	(0, 0.1)
Mutation rate	(0.01, 0.5)
Maximum nodes of a GP program	50
Function Set	{ +, -, ×, ÷ (Protected) }
Number of Trials	100

Table 2: Summery of the Genetic Programming Parameters

Bargaining models of ICRub85, INA and ASY are one-sided incomplete information problems. The following one is a two-sided incomplete information model.

3.5 No information - NoInfo

We study a problem where both players know nothing about the each other's preference. [Fudenberg et al. 1983] studies a similar bargaining problem that neither players know his opponent's the valuation to the object. The rules differ from NoInfo though. They prove that there are multiple equilibria for the two-sided incomplete information.

4 Experiments using a Co-evolution Algorithm

We had solved the complete information bargaining problem CRub82 by using a two-population co-evolutionary algorithm. The average of shares from agreements in experiments approximates Subgame Perfect Equilibrium [Jin & Tsang 2005]. The co-evolutionary system is then modified, mainly on the genetic programmes' terminal sets, to serve the purpose of investigating four incomplete information bargaining problems.

4.1 Experimental Design

A co-evolutionary system to evolve co-adapted strategies is implemented by means of Genetic Programming (GP). The players are able to adopt different strategies thus each player should have a population consisting of candidate strategies. Both populations employ the same values of genetic operators: the population size, the generation size, the crossover rate, the mutation rate and the method of selection as stated in Table 2. Each population evolves following Evolutionary Algorithms, whilst the fitness assessments are subject to the co-evolving opponent population at the concurrent evolutionary time.

The strategy representation and fitness function are described in [Jin & Tsang 2005]. A Genetic programm g_i is an individual in the population for the player i, which consists of constants, variables and functions. Its corresponding time-dependent strategy is $s(g_i) = g_i \times (1 - r_i)^t$, where r_i

Bargaining Problems	*Player 1's Information set*	*Player 2's Information set*
CRub82	$\{\delta_1, \delta_2\}$	$\{\delta_1, \delta_2\}$
ICRub85	$\{\delta_1, w_0, \delta_w, \delta_s\}$	$\{\delta_1, \delta_2\}$
INA	$\{\delta_1, r'\}$	$\{\delta_1, \delta_2\}$
ASY	$\{\delta_1, r_2\}$	$\{\delta_1, \delta_2\}$
NoInfo	$\{\delta_1, r_2\}$	$\{r_1, \delta_2\}$

Table 3: Information sets for the two players of the five bargaining problems. GP Terminal sets are the information sets added $\{1, -1\}$.

is the discount rate $\delta_i \equiv exp(-r_i)$. Such strategies have no memory and no learning ability during a bargaining process.

Game Fitness of a strategy $s(g_i)$, is the average utility of $s(g_i)$'s gains from the agreements with strategies in the co-evolving opponent's population J which is a set of n bargaining strategies, $j \in J$:

$$GF(s(g_i)) = \frac{\sum_{j \in J} u_{s(g_i) \to s(g_j)}}{n} \tag{6}$$

this game fitness is also the raw fitness of the genetic program. $u_{s(g_i) \to s(g_j)}$ is $s(g_i)$'s utility from an agreement with $s(g_j)$. These bargaining problems imply a hard constraint: $x_i \in (0, 1]$, any offer, counter-offer or a share of an agreement should be no more than the size of the cake. In other words, the value of a GP program should be within $(0, 1)$. Otherwise, a penalty $h(g_i)$ should be applied.

A GP program's overall fitness $F(g_i)$ is defined as:

$$F(g_i) = \begin{cases} GF(s(g_i)) + 3 \\ \quad \text{if } g_i \in (0, 1] \\ h(g_i) \\ \quad \text{if } else \end{cases} \tag{7}$$

where $0 \leq h(g_i) \leq 2$.

There are 100 trials for every game setting for every problem. Experiments of the corresponding game settings adopt the same random sequences.

4.2 Terminal sets

These five bargaining problems differ from each other on the players' information completeness about the other's time preferences. For the ICRub85, the player 1 with a probability ω_0, believes the player 2's actual discount factor is δ_w. The player 1's information set is therefore $\{\delta_1, \delta_w, \delta_s, \omega_0\}$. Added the size of cake 1 and the -1 to change the sign, the terminal set for the genetic programmes in the player 1' s population is $\{\delta_1, \delta_w, \delta_s, \omega_0, 1, -1\}$. ω_0 is set to be 0.5 in our experiments. Similarly, for ASY problem the player 1 has no information about the player 2' discount factor so his information set is $\{\delta_1, r_2\}$. He has to guess δ_2 from random.

Table 3 lists the information sets of both players of these bargaining problems. Information sets add $\{1, -1\}$ become

Exp No.	Game Setting δ_1, δ_2, δ_2'	*Experi Mean* $\bar{x_1}$	*PBE* x_1^*		
			V_s	x^{ω_0}	$1 - y^{\omega_0}$
E1	0.1, 0.1, 0.9	0.9372	0.1099	*0.9045*	0.9095
E2	0.1, 0.9, 0.1	0.1486	0.1099	0.9045	*0.9095*
E3	0.4, 0.1, 0.8	0.9431	0.2941	*0.9200*	0.9080
E4	0.4, 0.6, 0.3	0.4903	*0.5263*	0.7488	0.7753
E5	0.4, 0.9, 0.3	0.0989	0.1563	0.7488	*0.7753*
E6	0.5, 0.5, 0.1	0.6647	*0.6667*	0.9265	0.9074
E7	0.9, 0.1, 0.6	0.9804	*0.8696*	0.9736	0.9026
E8	0.9, 0.4, 0.6	0.8895	*0.8696*	0.8602	0.6559
E9	0.9, 0.8, 0.1	0.6322	0.7143	0.9736	*0.9026*
E10	0.9, 0.9, 0.3	0.4815	0.5263	0.9054	*0.7284*

Table 4: Experimental results of ICRub85 Problem. δ_2 is the actual value of the player 2's discount factor and δ_2' is the incorrect one. If $\delta_2 < \delta_2'$, then $\delta_2 = \delta_w$, else $\delta_2 = \delta_s$. ω_0 is the possibility of the player 1' initial belief of δ_2 being δ_w. In experiments, ω_0 set 0.5. The Italic numbers under PBE x_1^* are the PBE selected by game theoretic analysis for its corresponding game setting.

the terminal sets. In terminal sets, r_1 and r_2 are uniformly distributed random real variables, $r_1, r_2, r' \in (0, 1)$. r' is from an approximate Gaussian distribution with mean δ_2 and deviation 0.1 but limited to the constraint $0 < r' < 1$. The value of r' is generated by sampling Gaussian values until a value within (0, 1) is sampled. Variables r_1, r_2 and r' keep as constants once created. In the initial populations, for one genetic program, a random variable only has one value and in different genetic programs they can have different values. Mutation or crossover may bring new random values, but will not change the values of those existing.

5 Experimental Results

The results of ICRub85 will compare with its game-theoretic solutions. We do not know game theoretic solutions for INA, ASY and NoInfo, so compare their experimental results with a benchmark, the experimental results of CRub82 to see the differences.

5.1 Experimental Results of ICRub85

According to game analysis, there are at least three possible equilibria for a set of (δ_1, δ_2, δ_2'). Which equilibrium is selected depends on the relationship of ω_0 and ω^* and whether the actual δ_2 is δ_s. For a game setting, $\bar{x_1}$ is the mean of the player 1's shares from agreements by the best-of-generation individuals at the 300^{th} generations of 100 trials. The Table 4 shows experimental shares $\bar{x_1}$ of agreements for the player 1 and his all possible PBEs, namely V_s (5), x^{ω_0} (3) and $1 - y^{\omega_0}$ (4) . The Italic numbers are the PBEs by game theoretic analysis for their corresponding game settings in conjunction with the condition that $\omega_0 = 0.5$.

We define that the experiments chose an equilibrium if the absolute difference value of $\bar{x_1}$ to this equilibrium is less than 0.05. In the case of the absolute difference to all three equilibria are larger than 0.05, which are E5 and E9. The one has the minimum difference to $\bar{x_1}$ is the equilibrium chosen. In the E1, the x^{ω_0} and $1 - y^{\omega_0}$ have very small

Exp No	*Experi Mean* $\bar{x_1}$	*choose* x^{ω_0}
E1	0.9372	0.9045
E3	0.9431	0.9200
E7	0.9804	0.9736
E8	0.8895	0.8602

Exp No	*Experi Mean* $\bar{x_1}$	*choose* V_s
E2	0.1486	0.1099
E4	0.4903	0.5263
E5	0.0989	0.1563
E6	0.6647	0.6667
E9	0.6322	0.7143
E10	0.4815	0.5263

Table 5: Equilibria chosen by Experiments.

Game Settings (δ_1, δ_2)	*C Rub82* $\bar{x_1}$	*IC Rub85* $\bar{x_1}$	*INA* $\bar{x_1}$	ASY $\bar{x_1}$	NoInf $\bar{x_1}$
(0.1 , 0.1)	0.9226	0.9372	0.9438	0.9536	0.9536
(0.1 , 0.9)	0.1920	0.1486	0.1474	0.1414	0.1306
(0.4 , 0.1)	0.9987	0.9431	0.9456	0.9546	0.9546
(0.4 , 0.6)	0.5092	0.4903	0.4981	0.4982	0.4951
(0.4 , 0.9)	0.1444	0.0989	0.0999	0.1016	0.1033
(0.5 , 0.5)	0.6754	0.6647	0.6765	0.6763	0.6761
(0.9 , 0.1)	0.9989	0.9804	0.9836	0.9859	0.9859
(0.9 , 0.4)	0.9104	0.8895	0.8944	0.8974	0.8971
(0.9 , 0.8)	0.6378	0.6322	0.6178	0.6245	0.6484
(0.9 , 0.9)	0.5141	0.4815	0.4965	0.4901	0.4930

Table 6: The mean of the player 1's shares $\bar{x_1}$ from the agreements by the best-of-generation individuals at the 300^{th} generation of 100 experimental trials for each game setting. Game setting are the actual values of δ_1 and δ_2. The results $\bar{x_1}$s of CRub82 are slightly different from the data reported in [Jin & Tsang 2005] due to the use of different random sequences. Such difference doesn't affect the conclusion drawn.

difference 0.005. We set that $\bar{x_1}$ choose the x^{ω_0}. The Table 5 separates the experiments which choose V_s and those choose x^{ω_0}. The hypothesis that the $\bar{x_1}$s have 0 difference from equilibria chosen by experiments is accepted with 95% confidence (t-statistic value = -0.6771 and t-Critical value two-tail = 2.2621).

No experiments choose the $1-y^{\omega_0}$ equilibrium. The reason is that this equilibrium is not an efficient one because it reaches the agreements after one time interval. Both players spend bargaining costs for such delay.

It is an obvious tendency that if the actual $\delta_2 = \delta_w$, the experiments choose x^{ω_0} equilibrium shown in all such cases: E1, E3, E7 and E8. If the actual $\delta_2 = \delta_s$, the experiments choose V_s equilibrium in the all such cases: E2, E4, E5, E6, E9 and E10. These observations suggest that evolution reveals the type of the player 2 and selects the most efficient equilibrium under the player 1' initial belief to 2' type, being correct.

Experimental results do not match with the PBE chosen by the game theoretic analysis. Instead results exhibit a strong consistency with the PBE when the player 1 correctly guesses out 2' type which coincides the values of SPE for the CRub82.

	R^2 *Values*
ICRub85	0.99636
INA	0.99591
ASY	0.99597
NoInfo	0.99483

Table 7: The $\bar{x_1}$s of CRub82 is the input y range. The $\bar{x_1}$s for an incomplete information problem are the input of x range.

5.2 The Mean Shares of Agreements

Table 6 lists the experimental $\bar{x_1}$ for each problem. An obvious pattern displays: given a game setting, the $\bar{x_1}$s across the five problems are very close to each other. Regression Statistics of R^2 on the 5 sets of $\bar{x_1}$s yield significantly high values nearly 1, which demonstrates the strong linear relationship among them, see Table 7. The only difference among these problems are the players' knowledge concerning their opponents' preferences. So the information completeness do not attribute much to the similarity of solutions among five problems at least at the end of evolutionary process. Instead, such observations imply that the actual values of δ_1 and δ_2 applied to the utility functions $x_1\delta_2^t$ and $x_2\delta_2^t$ probably determine the bargaining outcomes. Information completeness has less importance on agreements in evolutionary environments. The deviations of 100 runs' x_1s for every setting are very small. This shows the stability of the systems at the end of co-evolving process. No player (population) prefers to withdrawal from equilibria (co-adaptation) even in presence of invaders (mutations).

When there is only one unique equilibrium solved by game theory analysis, evolution tends to converge to such equilibrium [Jin & Tsang 2005]. If there exist multi-equilibria for a problem, for instance ICRub85, the evolution probably converge to one of equilibria where $u_1+u_2 = 1$ holds (u_i is the utility of the player i receives from an agreement).

5.3 Average Bargaining Time and Efficiency of Agreements

The cake is divided most efficiently when the bargaining time $t = 0$, as the cake is solely earned by the two players, spending nothing on the bargaining costs. The longer time spends on bargaining, the more bargaining costs incurred and therefore the less efficiency of this agreement. t is an indicator of the efficiency of agreements given discount factors.

Table 8 records the average bargaining time $\bar{t}$ for reaching agreements. The bargaining time $\bar{t}$s of the game settings $(0.1, 0.1)$ $(0.4, 0.1)$ and $(0.9, 0.1)$ are all the 0 across all the five problems. It is also interesting to note that these three game settings all have $\delta_2 = 0.1$, although δ_1 varies from 0.1 to 0.9. Increasing the value of δ_2, the $\bar{t}$ correspondingly rise. This can be confirmed by comparing the $\bar{t}$ values inside the groups where each group has its δ_1 keeps unchanged: $\{(0.1, 0.1), (0.1, 0.9)\}$,$\{(0.4, 0.1), (0.4, 0.6), (0.4, 0.9)\}$ or $\{(0.9, 0.1), (0.9, 0.4), (0.9, 0.8), (0.9, 0.9)\}$. In addition,

Game Settings (δ_1, δ_2)	*C Rub82* $\bar{t}$	*IC Rub85* $\bar{t}$	*INA* $\bar{t}$	ASY $\bar{t}$	NoInf $\bar{t}$
(0.1 , 0.1)	0.00	0.00	0.00	0.00	0.00
(0.1 , 0.9)	0.14	0.20	0.22	0.20	0.23
(0.4 , 0.1)	0.00	0.00	0.00	0.00	0.00
(0.4 , 0.6)	0.00	0.02	0.02	0.01	0.02
(0.4 , 0.9)	0.24	0.02	0.01	0.01	0.03
(0.5 , 0.5)	0.00	0.02	0.01	0.01	0.01
(0.9 , 0.1)	0.00	0.00	0.00	0.00	0.00
(0.9 , 0.4)	0.00	0.01	0.00	0.01	0.01
(0.9 , 0.8)	1.35	1.77	1.91	1.76	1.48
(0.9 , 0.9)	3.74	3.19	3.82	3.66	2.96

Table 8: The Mean of the bargaining time $\bar{t}$ when the agreements reached by the best-of-generation individuals at the 300^{th} generation of 100 experimental trials for each game setting. Results of the five problems, each with 10 game settings are shown.

when both δ_1 and δ_2 are not large enough (less than 0.8 in experiments shown), the association of bargaining time $\bar{t}$ with the δ_1 is relatively small and the all $\bar{t}$s are less than 1 so in most cases, such agreements are reached at the time 0. Contrarily , when both δ_1 and δ_2 are large enough, a slight increment of one discount factor will prolong the bargaining time dramatically. Shown in the Table 8, when the δ_2 rises from 0.8 to 0.9 when $\delta_1 = 0.9$, the bargaining time $\bar{t}$ is almost doubled. When both players have very high discount factors, the bargaining process extends much longer. The efficiency of agreements decrease, but not in an extraordinary degree because of the players' very low costs on time [1].

Having found that $\bar{t}$s highly associate with the discount factors , we notice that for each game setting, it is hardly distinguishable from the type of players' information completeness. In summery, the length of bargaining time, so is the efficiency of agreements, is more likely to be related to the game parameters than the players' information completeness. It's not clear whether the bargaining time is sensitive to the bargaining procedure.

5.4 The evolutionary time to stabilize

Evolutionary Algorithms (EA) are used as a stochastic search method, exploiting the idea of natural evolution. The search process starts from random points scattered over the search space and gradually confines into the more promising spaces. Often precise and enough information about the problems helps a more efficient and effective search. Having the full knowledge about each other, the experimental results of CRub82 are expected to converge to equilibrium(s) more quickly and more precisely than of NoInfo problems do, for instance.

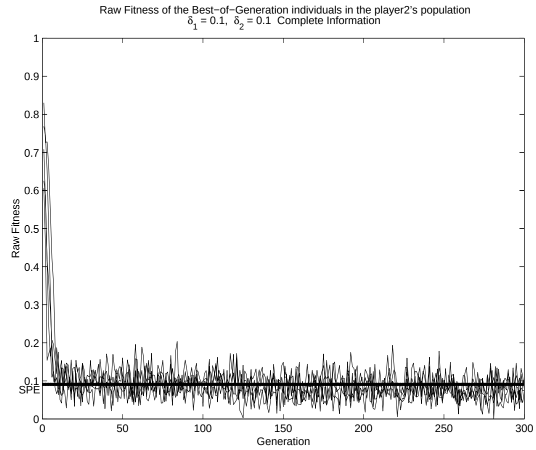

Figure 1: The raw fitness of the best-of-generation individuals in the player 2's population of Complete Information bargaining problem CRub82. 5 trials are shown. $\delta_1 = 0.1$, $\delta_2 = 0.1$

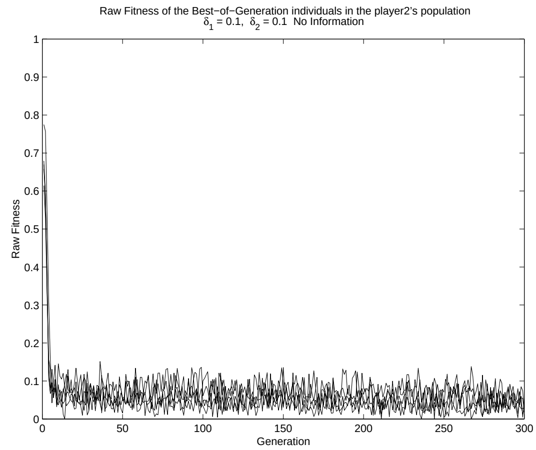

Figure 2: The raw fitness of the best-of-generation individuals in the player 2's population of a NoInfo bargaining problem. 5 trials are shown. $\delta_1 = 0.1$, $\delta_2 = 0.1$

Two sets of examples are given to address this concern. The first set includes a CRub82 bargaining problem and a NoInfo problem, both with the same discount factors ($\delta_1 = 0.1$, $\delta_2 = 0.1$) and the second set has a CRub82 problem and a NoInfo problem with the same discount factors ($\delta_1 = 0.9$, $\delta_2 = 0.4$). Five trials are displayed on every figure. One set of experiments use the same sets of random sequences. Figure 1 and 2 illustrate the raw fitness of the best-of-generation individuals for the player 2' populations. In Figure 1, both players have the complete information. The raw fitness are around $0.8 \sim 0.9$ at the early beginning, quickly dropping down to the area $0.05 \sim 0.15$ and fluctuate in the proximity of SPE $x_2^* = 0.0909$. Surprisingly, figure 2 shows similar phenomenon, although neither player has any information about the other. In this exam-

[1]For example, the $\bar{t}$ of the game setting $(0.9, 0.9)$ is about 3, then the average bargaining cost of a strategy in the player 2' population is therefore $(1 - 0.9)^3 = 0.001$. The $\bar{t}$ of $(0.1, 0.9)$ is 0.20, then the average cost of a strategy in the player 2' population is therefore $(1 - 0.9)^{0.2} = 0.63$. This example shows that even the average bargaining time of (0.9, 0.9) is 15 times longer than that of the (0.1, 0.9), the average bargaining cost of a player 2' strategy in (0.9, 0.9) is only 0.16% of the cost of a player 2' strategy in (0.1, 0.9).

ple, the knowledge of the opponent has little effect on the time for finding out the co-adaptive strategies and stabilization. In another example, Figure 3 and Figure 4 illustrate the fitness of the best-of-generation individuals in the player 1' populations, for the problem of the CRub82 and of the NoInfo, respectively. It is interesting to see that from as early as the first generations, the five trials of NoInfo leap to the point which stabilizes for 300 generations. Some trials for the CRub82, however, spend a significantly longer time to reach it: gradually increase to the SPE in a series steps. It probably is caused by the fact that the large number of random values as terminal nodes in the genetic programmes for the NoInfo problem provide a greater diversity, making the search space much larger. In contrast, the terminal set of CRub82 only has δ_1 and δ_2, although they are precise.

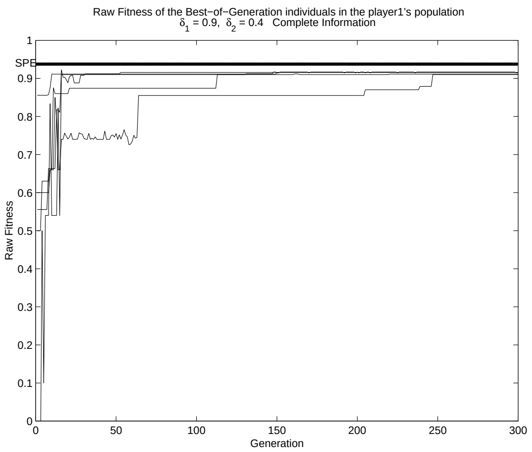

Figure 3: The raw fitness of the best-of-generation individuals in the player 1's population of a Complete Information bargaining problem CRub82. 5 trials are shown. $\delta_1 = 0.9$, $\delta_2 = 0.4$

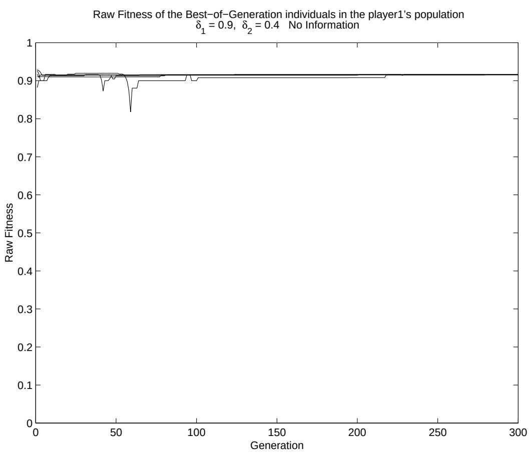

Figure 4: The raw fitness of the best-of-generation individuals in the player 1's population of a NoInfo bargaining problem. 5 trials are shown. $\delta_1 = 0.9$, $\delta_2 = 0.4$

6 Discussion

The common sense of business tells us that if a player has more information related to the bargaining outcomes he should attain more advantages. However, the experimental results by co-evolutionary algorithm demonstrate no apparent indication of that the amount or precision of knowledge obviously affects bargaining outcomes on partitions of the cake in agreements. From the viewpoint of game-theoretic analysis, bargaining problems under one-sided or two-sided incomplete information imply delay and inefficiency [Fudenberg et al. 1985]. But experimental results on bargaining time provide no evidence of obvious delays due to the incomplete information settings.

Equilibria selected by experiments are always the most efficient ones for the evolutionary systems. They are reached because, through the selection and variation mechanism (genetic learning), the information that exist but unknown by the other population is induced via fitness evaluations and then embedded in the individuals at a genetic level. This means that the co-evolutionary process prefers (select for reproduction) those genetic programm individuals that act *as if* they have fully informed about all aspects of the game, in that these individuals perform better.

If the missing information is dynamic, in other words, the preference of the other player is unknown and changing randomly within a range over bargaining time or over evolutionary time, then the co-evolutionary process may select for survival those individuals that act rationally (most efficiently in terms of fitness) according to the fact that the behaviors of their opponents are really unknowable within the specified range of variability. [Ordover & Rubinstein 1983] describes such a dynamic missing information game. [Billings et al. 2002]'s Porker is also a dynamic missing information one as (several) participants constantly update their preferences.

On the other hand, although the experiments have found one of the Pareto efficient and stationary equilibriums for a game that has no game-theoretic solutions, other possible equilibriums have been discarded. This feature limits the applications of EA to such games which need to stress the importance of more than one feasible solutions. Moreover, EA' ability to discover private information harms its potential applications on observing special features of some one-off games where the chance and time for learning are extremely limited and random elements play a strong role.

7 Conclusion

In summery, this work investigates experimental results by a co-evolutionary algorithm for incomplete information bargaining problems. As observed, the experimental results of the four incomplete information problems demonstrate linear correlation among them as well as results of the complete information problem, in terms of the shares of agreements and efficiency of agreements, irrespective of players' information completeness. The length of bargaining time is more likely to be related to bargaining parameters rather than the state of player(s)' information is. In particular, experiments of ICRub85 choose the equilibria which ensure the high efficiency of agreements and the player(s)' initial belief to his opponents being correct. Moreover, it is not necessary to spend much longer time for evolutionary populations to find out such equilibria in absence of full knowledge.

Solutions by co-evolution stabilize at (one of) the most efficient and stationary equilibria. This approach therefore, is capable of providing reasonably good solutions for those bargaining problems that have no game theoretic solutions now due to their complexity. Unlike the game theoretic method that requires considerable efforts to solve bargaining problems with a minor change, the co-evolutionary system is reusable for variants of bargaining problems. This work also presents evidences of a general phenomenon that the co-evolving populations eventually uncover the static private information and treat the problems as full informed. It is necessary to check other games to ensure the generality of this finding.

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